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SOME PROPERTIES OF THE VAN ALLEN RADIATION

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The Van Allen radiation belt (energetic particle radiation trapped in the geomagnetic field) has been shown[1-4] to consist of a hard proton component centered at about 10×10^3 km from the earth's magnetic axis (the proton belt) and an electron component centered at about 22×10^3 km from the earth's magnetic axis (the electron belt). The electron belt extends through the region occupied by the proton belt. It is generally accepted that the proton belt is produced by the decay of cosmic-ray albedo neutrons,[5-8] although some doubt has been expressed on this point.[9] The electron belt is usually held to be of solar origin.[10] The purposes of this Letter are twofold: first, to point out that the results of the observations of the outer zone of the Van Allen radiation belt made with the Explorer IV and Explorer VI satellite systems are inconsistent with the solar injection hypothesis; and second, to show that the electrons released in the decay of cosmic-ray neutron albedo may represent a satisfactory source for the outer zone. Source strength and trapping lifetime are not discussed.[11]

An illustrative sketch of particle flux vs radial distance in the equatorial plane is shown in Fig. 1. This sketch is based principally on the measurements made by Simpson and his co-workers.[12] Hydromagnetic scattering probably limits the radial extent of the proton belt.[13] The measurements show that the electron belt consists of two regions separated by a relative minimum near 19×10^3 km from the geomagnetic axis. We know of no satisfactory explanation for this gap in the electron belt other than the one offered by the Capetown anomaly (a region several thousand kilometers in extent, centered southwest of Capetown, South Africa, and having an abnormally weak magnetic field strength).

It has been pointed out that the Capetown anomaly lowers the mirror altitude of the trapped radiation reflected near the anomaly by about 1000 km.[14] Since the atmospheric density is almost constant above about 1300 km where hydrogen predominates and increases very rapidly below this altitude where heavier atmospheric constituents predominate,[15] we should expect radiation mirroring

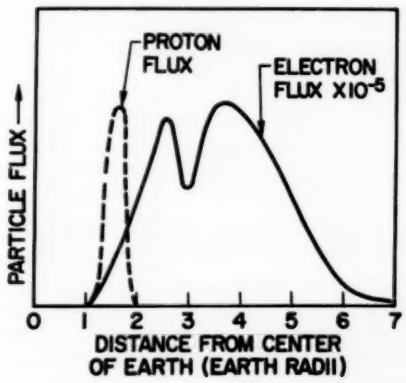

FIG. 1. Illustrative sketch of the Van Allen radiation belt particle flux vs radial distance in the equatorial plane. This sketch is based principally on data presented by Simpson et al.[12] The electron belt undergoes changes during magnetic storms which produce large variations in the counting rate of instruments flown through the belt.

below about 1300 km to be rapidly removed from the radiation belt. As the trapped radiation drifts around the earth's magnetic axis, a zone of relatively low radiation intensity will be developed at the magnetic latitude of the Capetown anomaly because that part of the Van Allen radiation which normally mirrors at altitudes up to 2300 km will be removed by atmospheric scattering at the lower mirror altitude over the Capetown anomaly. The part of the radiation belt affected by the anomaly has been determined from the surfaces given by the two adiabatic invariants (magnetic moment and integral invariant) of a charged particle in the earth's field as calculated by Vestine and Sibley[16] using the 48-term harmonic analysis of the geomagnetic field. With these data plus data on the altitudes of surfaces of constant field strength,[16] the effect of the Capetown anomaly is illustrated in Fig. 2. In this figure, mirror altitude is plotted against longitude for 3 different shells (the surface generated by a particle as it drifts arount the magnetic axis) as defined by the adiabatic invariants. It is apparent from this figure that the greatest decrease from a "normal" mirror altitude occurs over the Capetown anomaly for the field line or shell which crosses the equatorial plane at 17×10^3 km from the earth's center. Adjacent

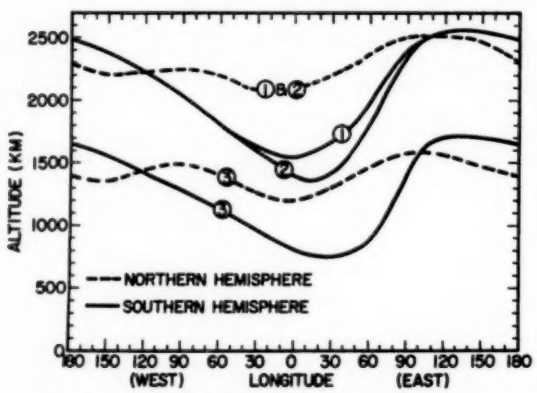

FIG. 2. The altitude of a given mirroring field, B_m, along 3 different traces determined from integral invariant calculations for magnetic field lines which extend a distance b from the earth's center in the equatorial plane.

Trace (1): $B_m = 0.18$ gauss, $b = 13\times10^3$ km.
Trace (2): $B_m = 0.20$ gauss, $b = 17\times10^3$ km.
Trace (3): $B_m = 0.30$ gauss, $b = 28\times10^3$ km.
Note that the average mirror point altitude for trace (2) in the northern hemisphere is lowered by about 1000 km over the Capetown anomaly. Traces (1) and (3) are not so severely affected.

shells on either side of this one show a less pronounced effect. Further, it is expected that the sharpness of the anomaly is underestimated because the spherical harmonic analysis contains too few terms to describe its features adequately.[18] This view is supported by recent magnetic field measurements made by Heppner et al., with the Vanguard III satellite.[17] These measurements show that at altitudes of 2000 - 3000 km, the Capetown anomaly is much sharper and slightly deeper than predicted. Thus, we may conclude that the effect of the Capetown anomaly is much more localized than shown in Fig. 2. It therefore appears that the observed gap in the electron belt is in satisfactory agreement with the location of the Capetown anomaly.

As we have said, the Capetown anomaly can lead to the rapid removal only of electrons with mirror points below 2300 km altitude which will be reflected below 1300 km over the anomaly. Electrons with high mirror points will not be affected and will be responsible for a counting rate in the region where the gap is observed. The electron source, therefore, if reasonably isotropic, must inject electrons predominantly at low altitudes so as to assure low mirror points. This is quite a severe requirement. It seems obvious that particles of solar origin would not satisfy it. Electrons from neutron decay, on the other hand, do have the desired property of mirroring principally at low altitudes. Hess[8] finds that the neutron decay density near the earth varies approximately as $r^{-4}(1+3\sin^2\lambda)$, where λ is the latitude. This injection distribution leads to a flux decrease of about 10% near the equator if we use an altitude of 1300 km as the lower cutoff for injection but retain only particles mirroring above 2300 km. On the basis of this model we predict that the depth of the gap in the electron belt will show an altitude dependence: the gap will be more pronounced at low altitudes than near the equator on the magnetic field shell that passes through the Capetown anomaly.

The spectrum of the electrons resulting from neutron decay is peaked near 300 kev. The electrons which form the radiation belt are "cooled" by electron-electron collisions[11] to the observed E^{-5} spectrum.[2] These interactions will also affect the depth of the gap as measured by detectors of different thresholds.

We turn finally to a discussion of the magnetic-storm-induced fluctuations in the radiation belt.[12,18] These fluctuations (during which the counting

rate in the electron belt may change by 10^4) are the principal evidence cited in support of the solar origin hypothesis of the trapped electrons. There are four difficulties with the solar injection hypothesis: (1) Solar particles could most probably be injected during the active phase of a magnetic storm, when the dipole character of the earth's field is disturbed. Yet, at this time, a decrease in counting rate is observed. That is, when additional electrons could be injected, they are not. (2) After the active part of a storm is over (recovery phase—low K index) the observed counting rate increases by several orders of magnitude. However, this is the time when the geomagnetic field is restored so that the entry of any but cosmic-ray particles into the field is most unreasonable. Thus, we argue that the counting rate increase occurs too late for solar injection to have been effective. (3) The particles have a lifetime of a year or more in the very rare atmosphere at high altitude. This lifetime is in accord with the fact that the intensity of the belt is stable during magnetically quiet periods. Nevertheless, with the solar injection hypothesis it is proposed that additional electrons are injected during a magnetic storm and that most of them disappear with a lifetime of a few days. This short lifetime seems inadmissible. (4) As discussed earlier, the existence of the gap in the electron component, and especially its persistence during and after magnetic storms, seem impossible to reconcile with the solar injection hypothesis.

The 10-kev solar wind protons, whose injection into the earth's field gives rise to the stresses that produce the magnetic field decrease during the main phase of a magnetic storm, have a trapping lifetime that is comparable to the time required for the enhanced counting rate to return to prestorm values. These protons are removed by charge exchange with the neutral hydrogen telluric corona.[19] We propose, therefore, that it is the magnetic field changes rather than changes in the number of trapped particles that give rise to the varying counting rates. There are two mechanisms by which this may happen. First, since the field changes are slow compared to the electron cyclotron frequency, the displacement of field lines will cause an accompanying displacement of particles. Many particles may therefore appear where only few had been before. Second, changes in the field intensity will result in an energy change of the particles; because the radiation counters have a threshold, the measured flux will be strongly affected. Both mechanisms can cause large changes in the counting rate, the first because the particle density varies rapidly with position, the second because the energy spectrum is very steep (e.g., an integral spectrum[2] as steep as E^{-5}). It seems clear that an appreciable fraction of the magnetic storm energy can be temporarily given to the radiation belt by the mutual inductance between the ring currents: (1) the solar wind protons and (2) the Van Allen belt electrons.

There are some incompletely treated problems that must be resolved before we can claim a complete understanding of the radiation belt. The source strength, the trapping lifetime, and the details of the magnetic storm effects[20] are the most important. Still, we feel that the present analysis leads to a fairly satisfactory picture in which the entire Van Allen radiation belt is due to neutron decay: The decay of fast neutrons produces the proton belt while the electron belt is attributed to the large number of slow neutrons which decay relatively near the earth.

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VARIATION WITH TEMPERATURE OF THE ENERGY OF RECOIL-FREE GAMMA RAYS FROM SOLIDS*

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The 14.4-kev γ ray emitted without recoil by 0.1-μsec Fe^{57} in metallic iron[1-4] excited great interest as the most precisely defined electromagnetic frequency yet discovered. It may be adequately well defined to allow measurement of the influence of a gravitational potential on frequency[5] and of other small effects hitherto beyond the sensitivity available in the laboratory. As a preliminary step in the operation of an experimental system designed to measure the gravitational effect, we have been making tests to find out whether other influences than the one intended might lead to systematic errors by introducing important frequency shifts not taken into account.

So far the largest such effect found is that of temperature. That temperature should influence the frequency exactly as we observe is very simply explained. Thermally excited vibrations cause little broadening through first order Doppler effect under the conditions obtaining in the solid because the value of any component of the nuclear velocity averages very nearly to zero over the nuclear lifetime. The precision of the γ ray of Fe^{57} requires the second order Doppler effect also to be considered. A shift to lower frequency with increased temperature results from this because the also well-defined average of the square of the velocity of the particle increases in direct proportion to the average kinetic energy. As a consequence one would expect a temperature coefficient of frequency in a homogeneous solid,

$$(\partial\nu/\partial T) = -\nu C_L/2Mc^2,$$

where C_L is the specific heat of the lattice and M is the gram atomic weight of iron. In the high-temperature classical limit where $C_L = 3R$,

$$(\partial\nu/\partial T)_{T\to\infty} = -2.44\times10^{-15}\nu \text{ per } ^\circ\text{K}.$$

At lower temperatures one would expect a coefficient reduced by the value of the appropriate normalized Debye specific heat function. For iron, at 300°K one should find about 0.9 times, and at 80°K about 0.3 times, the above classical value.

The temperature dependence has been measured by counting the γ rays from our 0.4-curie Co^{57} source transmitted through enriched Fe^{57} absorbing films (0.6 mg Fe^{57}/cm^2). The Co^{57} of the source is distributed in about 3×10^{-5} cm thickness below the surface of a 2-in. diameter iron disk, made in the manner described earlier.[1] Small frequency shifts that result when the source and absorber are held at different temperatures were measured by using a transducer to move the source sinusoidally at ten cps toward and away from the absorber at a peak speed of about 0.01 cm/sec. A gate pulse and mercury relays were used to make one counter record during 25 milliseconds of the modulation period symmetrically disposed about the time of maximum velocity toward the absorber. Another

counter recorded the corresponding counts with the source going away from the absorber. The difference of the counts in the two registers should be proportional to the relative frequency shift of the absorber and source for shifts small compared to the line width. Quantitative knowledge of the parameters of the system that are involved in determining the constant of proportionality is rendered unnecessary by adding through a clock-driven hydraulic system a continuous relative motion of 6.3×10^{-4} cm/sec directed oppositely during each of the two halves of the time for a given datum point. In this way the sensitivity to frequency shift originating in the Doppler effect is measured simultaneously with the shift sought. The algebraic sum of the counting rate differences for the two halves of the run are proportional to the shift and the difference to the sensitivity.

The shift at liquid nitrogen relative to room temperature is comparable to the line width and for that point the two counting rates were recorded at a series of values of the sinusoidal modulation amplitude. From these a value of the shift and of the apparent line width could be obtained although difficulties of calibration under the conditions of operation have contributed strongly to the uncertainties. There is evidence that the line appears to broaden with such a temperature difference by perhaps a factor of 2.3 which might be evidence that the hyperfine structure splittings are temperature sensitive to some extent, as must be expected.

The data are plotted in Fig. 1. A solid line representing the effect expected with a Debye temperature of 420°K is also drawn. The agreement can be regarded as an experimental demonstration of the second order Doppler effect using thermal velocities rather than a centrifuge. It might be remarked that crystalline anisotropy might make this source of high velocities useful for experiments to the end of detecting such spatial anisotropies as might accompany ether drift or an inertial frame.

The temperature sensitivity at room temperature [experimentally $(-2.09\pm0.24)\times10^{-15}$ per degree C, theoretically -2.21×10^{-15} per degree C] is highly relevant to the interpretation of data

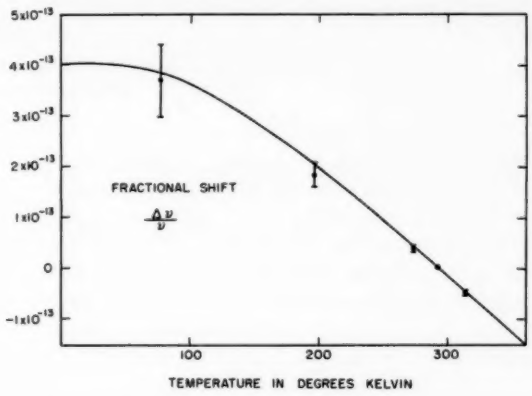

FIG. 1. Fractional shift of energy of 14.4-kev gamma-ray absorption of Fe^{57} vs absolute temperature of the metal. The solid line is derived from assuming a Debye temperature of 420°K.

from our experiment on the effect of gravitational potential. A temperature difference of 1°C between the top and the bottom of our 22-meter tower would result in a shift about equal to that predicted by the principle of equivalence. For smaller height differences correspondingly smaller temperature differences would be required. It is now clear that correction for or control of the temperature difference and perhaps other parameters must be included in the instrumentation of experiments intending to utilize the extreme frequency discrimination available with gamma rays of this type.

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ELECTRON SPIN RESONANCE IN BISMUTH AND ANTIMONY

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(Received February 17, 1960)

Spin resonance of conduction electrons has been observed previously in the alkali metals[1] with a g factor approximately that of the free electron, $g=-2$. More recently[2,3], observations and calculations of the g factor for conduction electrons in the semiconductor InSb have shown it to be very large ($g=-50$) because of the small energy gap and large spin-orbit coupling involved. The same situation—small energy gap and large spin-orbit coupling—exists in the semimetals bismuth and antimony and, for the case of bismuth, Cohen and Blount[4] have calculated the g factor in terms of the effective mass tensor. It is expected that their theory should be equally valid for antimony. This Letter reports the experimental observations of spin resonance in these materials and generally confirms the predictions of Cohen and Blount.

The measurements were made on crystallographically oriented plane surfaces of zone-refined single-crystal samples which had been acid polished. The experimental arrangement was such that the sample formed one end of a microwave cavity. A signal proportional to the variations in the Q of the cavity, and therefore to variations in the power absorption coefficient of the sample surface, was observed as a function of magnetic field. The measurements were made at 72 kMc/sec and 1.3°K. Details of the apparatus have been reported elsewhere.[5]

The results for antimony were obtained with the magnetic field perpendicular to the plane of the sample and with circularly polarized radiation incident upon it. The bisectrix axis of the crystal was oriented parallel to the magnetic field. A plot of power absorption vs magnetic field, shown in Fig. 1, exhibits a sharp resonance at 1.14 kilogauss and a sharp rise preceded by a dip and with some structure upon it at 2.28 kilogauss. An upper limit on the line width of the sharp resonance at 1.14 kilogauss is 30 gauss. It is felt that this line could not be attributed to cyclotron resonance because its sharpness indicated too large a relaxation time. Furthermore, neither this line nor the dip and other structure on the sharp rise are understandable in terms of the theory of such a curve taking account of cyclotron resonance, while the sharp rise itself is understandable in these terms as a "dielectric anomaly."[5]

The bismuth data shown in Fig. 2 were obtained using linearly polarized radiation with the static magnetic field in the plane of the sample and parallel to the trigonal axis. In the top curve, the microwave H field and the static H field are perpendicular, while for the bottom curve they are parallel to one another. The appearance of resonances in the top curve and not the bottom indicates a magnetic dipole transition and convinces us that the sharp resonances observed arise from spin flips. These effects have also

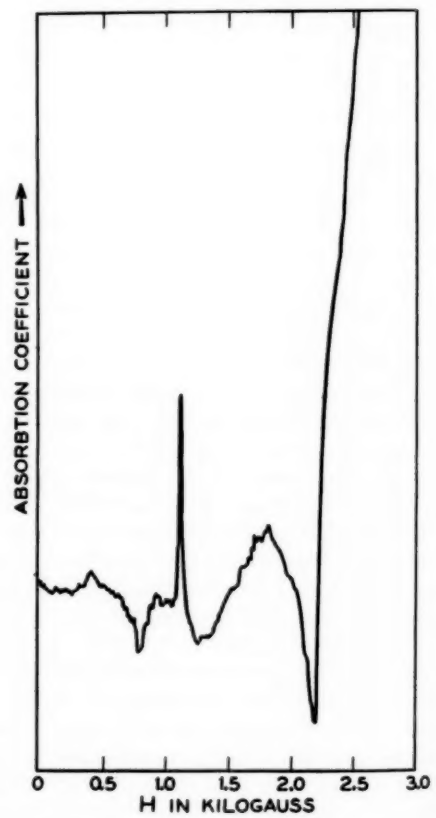

FIG. 1. Microwave absorption in antimony with circularly polarized radiation. The static field is normal to the sample surface and along a bisectrix axis.

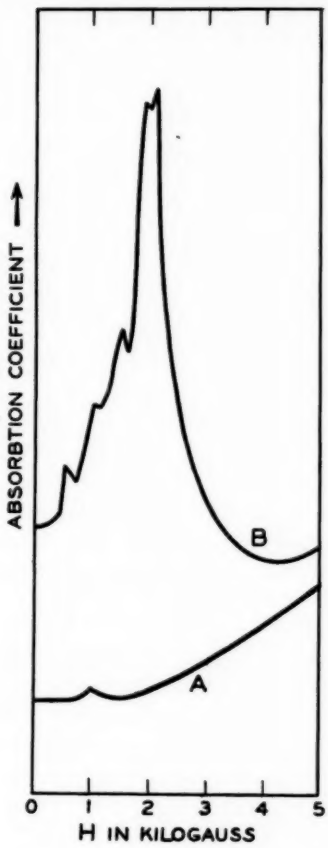

FIG. 2. Microwave absorption in bismuth using plane polarized radiation. Curve A has the microwave H field parallel to the static H field and curve B has them perpendicular to one another. The static field is along a trigonal axis.

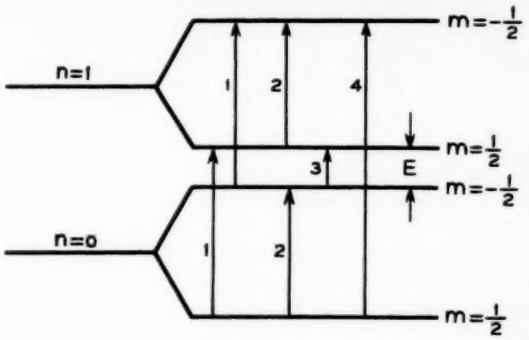

FIG. 3. Energy level diagram showing two Landau levels and the effect of spin splitting for a group of conduction electrons in bismuth and antimony.

been seen with the magnetic field along twofold and bisectrix axes in bismuth.

The essential result of Cohen and Blount's theory is that the spin effective mass m_S [defined by $m_S/m = 2/g$, where m is the free electron mass] should be equal to the orbital effective mass m_0 when the orbital effective mass is sufficiently small. An energy level diagram showing the spin splitting nearly equal to the orbital splitting is shown in Fig. 3. Higher Landau levels ($n = 3$, 4, etc.) are similar to the first two. Type 1 transitions are the usual cyclotron resonances, and type 2 are pure spin resonances. Types 3 and 4 combine an orbital transition with a spin flip and have not previously been considered. The theory indicates that the energy difference marked E can vary with orientation from zero to nearly the Landau spacing, depending upon the mass parameters involved. Our data have been tentatively interpreted in terms of this scheme.

Electron and hole mass parameters obtained from cyclotron resonance in bismuth[5] predict resonances of type 1 at 2.1 and 1.7 kilogauss, respectively. The peak observed at 2.0 kilogauss is interpreted as a type 2, or spin, resonance, and the close agreement between these fields suggests that E is very small in this orientation. The fact that the experimental peak appears doubled is thought to be due to slight sample misorientation. There are three other peaks, one of which is tentatively believed to arise from hole spin resonance and the other two from type-4 transitions, one for electrons and the other for holes. The exact positions of the resonances are in some doubt, both because of their close spacing and because the resonance field is shifted from the peak, according to the theory of conduction electron spin resonance.[1,6] In spite of this uncertainty, however, the data lend considerable credence to the above interpretation. Transitions of type 3 as well as of types 2 and 4 have been observed in bismuth at other orientations. The observation of type-3 transitions implies that $E \neq 0$ is bismuth, at least in some orientations, and the value of this separation determined from our data as taken with the magnetic field along a dyad axis agrees with a splitting seen earlier in de Haas-van Alphen type oscillations in the electronic specific heat by Kunzler and Boyle.[7] The g factor observed with the field along this axis is about -200 for the low-mass electrons. The resonance peak at 2.1 kilogauss with the magnetic field along a threefold axis suggests a g factor of about -30 for electrons.

The antimony data show cyclotron resonance

at 2.28 kilogauss, a field which is obtained by extrapolating the sharply sloping portion of the curve to zero absorption, and which corresponds to an electron cyclotron mass of $0.088m_0$. This method for identifying the cyclotron resonance is justified in reference 5. The sharp resonance at 1.14 kg is identified as a type-4 transition and the fact that it is at exactly half the cyclotron resonance field indicates that E is zero within the accuracy of the experiment. A dip preceding the sharp rise at cyclotron resonance and a kink in the slope of the sharp rise indicates that the pure spin resonance line is present at exactly the cyclotron frequency, as is expected if $E = 0$. The g factor for this resonance is 23.

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SPIN SPLITTING OF THE LANDAU LEVELS IN BISMUTH OBSERVED BY MAGNETOTHERMAL EXPERIMENTS

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In two previous oral presentations[1] we have described a new type of manifestation of the oscillatory dependence on magnetic field of the density of states of an electron gas at the Fermi surface. Specifically, there is a periodic (in $1/H$) dependence of the electronic heat capacity on magnetic field as well as an orientation dependence at constant field if the Fermi surface is nonspherical. The previous reports described some observations that have been made on a single crystal of bismuth and it was pointed out that this type of experiment offers certain advantages in sensitivity, simplicity, and interpretation over other types of magneto-oscillatory experiments. In this Letter new data on bismuth at larger magnetic fields are presented which we interpret to give information on the spin splitting of the Landau levels.

Only an essential outline of the experiment necessary for this report will be given here. However, a more complete description and the detailed experimental results are in the course of preparation. The oscillatory component of the total heat capacity is small (usually less than 0.1% in bismuth) at 1.4°K. Therefore, direct heat capacity observations are impractical and the approach adopted is to observe the changes in heat capacity as a function of the magnitude or orientation of the magnetic field. Experimentally these appear as temperature changes under essentially adiabatic conditions. This is accomplished by mounting the oriented bismuth crystal in a suitable powder[2] selected to provide a sample to helium bath thermal relaxation time of several minutes. The temperature differences are continuously recorded by using a carbon thermometer in good thermal contact[3] with the bismuth crystal.

These cooling peaks occur when the bottom of a Landau sub-band passes through the Fermi surface and result from the increased density of states at the bottom of each band and hence an increased entropy of the electron gas. With a nonspherical Fermi surface this condition depends on both magnitude and direction of the magnetic field as given by the following equation:

$$E_f = (n_f + \tfrac{1}{2} \pm \Delta)\hbar eH/m^*c. \quad (1)$$

Δ is a parameter that determines the spin splitting and is given by

$$\Delta = m^*g/4m, \quad (2)$$

where m^* and g are, respectively, the cyclotron mass and the spectroscopic splitting factor for the pertinent orientation and m is the free electron mass. n_f is the quantum number of the Landau level, E_f is the Fermi energy, and all other symbols have their usual significance.

In cases where $m^* \ll m$, using a free spin g factor of 2 leads to a value of $\Delta \simeq 0$. Examination of Eq. (1) then shows that a plot of n_f against $1/H$ should give an intercept near a half-integral value of n_f. However, the de Haas–van Alphen susceptibility data of Dhillon and Shoenberg[4] show that the intercept is considerably different than 1/2. Our magnetothermal data[1] show that the intercept is within a few percent of being an integer when the magnetic field is in a low-mass direction. This is consistent with Cohen and Blount[5] who have shown theoretically that for the low-mass directions, the g factor in bismuth is

$$g \simeq 2m/m^*, \qquad (3)$$

which makes $\Delta \simeq 1/2$. The effect of the various factors discussed above on the separation of the Landau levels is illustrated schematically in Fig. 1.

When we consider data at high enough magnetic fields so as to include one of the $n=0$ levels, the splitting of the sub-bands can be seen as shown in Fig. 2. The value of Δ can be derived from the pair of peaks near 15 kilogauss. Since the separation of the peaks varies as H^2, the lower field peaks are not resolved. The resolution is somewhat better on rotational diagrams. Figure 3 is a rotational diagram covering 180° taken at 15.5 kilogauss with the magnetic field perpendicular to the threefold axis. The pronounced peak with the field along the binary axis at 0° (and at ± 60°) is the 15.5-kilogauss peak of Fig. 2 and

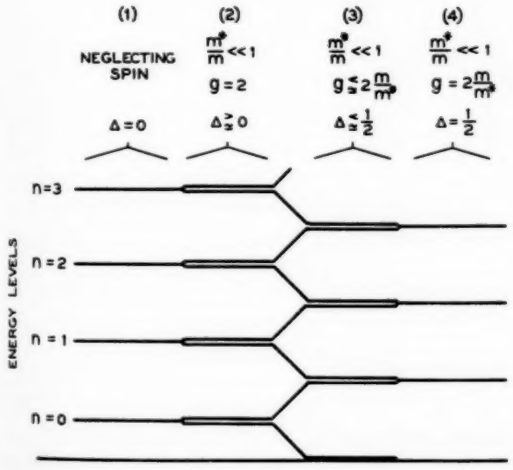

FIG. 1. Effect of various situations on the spin splitting of Landau levels for a given value of crystal momentum along the magnetic field. (1) Spin is neglected and therefore $\Delta = 0$. (2) $g = 2$ is used for electrons of mass much smaller than the free electron mass. (3) The g factor for electrons in bismuth is approximately equal to but less than $2m/m^*$. (4) $g = 2m/m^*$ and $\Delta = 1/2$.

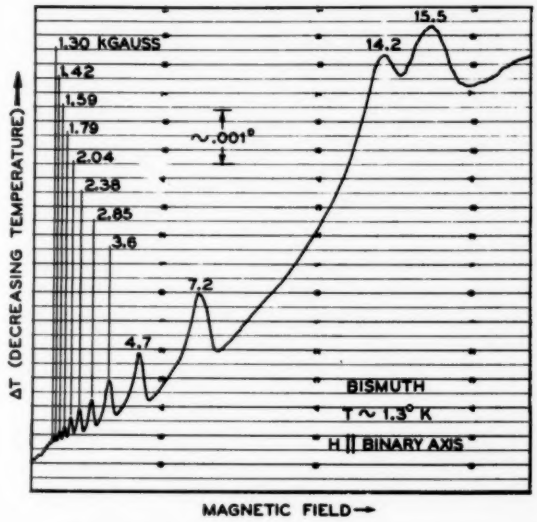

FIG. 2. Cooling peaks as a function of magnetic field for the field along the binary axis. The values of magnetic field noted for each peak are from rotational diagrams which allow more precise assignments. The spin splitting is evident for the two peaks near 15 kilogauss but not for peaks at lower fields.

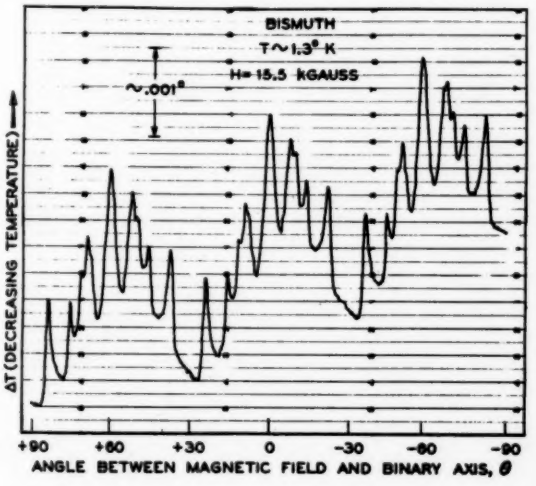

FIG. 3. Rotational diagram about the threefold axis at 15.5 kilogauss showing angular dependence of cooling peaks.

represents contributions from four equivalent groups of electrons. Each of the major peaks at $\pm 8°$ originates from two of the four groups of electrons that are responsible for the 14.2-kilogauss peak of Fig. 2. The other peaks in Fig. 3 have been unambiguously accounted for but are not pertinent to the present report. A series of rotational diagrams at fields near the 7.2-kilogauss peak of Fig. 2 provides just enough resolution to demonstrate the splitting of that pertinent level.

From the positions of the resolved peaks of Fig. 2, 14.2 and 15.5 kilogauss, and assuming that Δ is less than 1/2, we obtain

$$\Delta = 0.46, \qquad (4)$$

for the low-mass electrons and the field along the binary axis.

A value of Δ different from 1/2 suggests that an effect should be observed in cyclotron resonance and spin resonance observations. Smith, Galt, and Merritt[6] have observed transitions which they interpret in a manner consistent with these ideas.

When the 14.2- and 15.5-kilogauss peaks are included on the plot of $1/H$ vs peak number of the unresolved peaks at the lower fields, one would expect that one peak would fall about 4% above and the other 4% below the curve. However, it is found that the actual values for the magnetic fields are about 4% higher for the peaks. This is well outside the experimental uncertainty of the positions of the peaks at lower field ($\pm 1\%$) and is ascribed to oscillations in the Fermi energy which become large for the last level.

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[1]J. E. Kunzler and W. S. Boyle, Bull. Am. Phys. Soc. 4, 168 (1959); J. E. Kunzler, Bull. Am. Phys. Soc. 4, 319 (1959).

[2]J. E. Kunzler (to be published).

[3]W. S. Boyle and K. F. Rodgers, J. Opt. Soc. Am. 49, 66 (1959).

[4]J. S. Dhillon and D. Shoenberg, Phil. Trans. Am. 248, 1 (1955).

[5]M. Cohen and E. I. Blount, Phil. Mag. (to be published).

[6]G. E. Smith, J. K. Galt, and F. R. Merritt, preceding Letter [Phys. Rev. Letters 4, (1960)].

F-BAND STRUCTURE IN CESIUM BROMIDE

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(Received February 8, 1960)

The *F* center in NaCl-type alkali halides has been extensively studied,[1] but comparatively little is known of the *F* center in CsCl-type alkali halides. It has been reported recently that the principal optical absorption bands in two CsCl-type alkali halides, CsCl and CsBr, show component band structure at liquid helium temperature.[2] It is the purpose of this Letter to report current results obtained on CsBr which indicate that this structure is due to a single center, most likely the *F* center.

Figure 1 shows the optical absorption spectra of two CsBr single crystals, one a Harshaw crystal, additively colored with potassium,[3] and the other an MIT-grown crystal,[4] x-ray irradiated for 90 minutes (43 kvp and 20 ma) at room temperature. The spectra over the range 350 to 750 mμ are shown at room temperature, curves (a) and (c), as well as at liquid helium temperature, curves (b) and (d). The structure in the principal absorption band of curve (b) is shown in detail in Fig. 2 over the range 550 to 700 mμ. It is clear that the absorption spectra in both the additively colored crystal and the x-rayed crystal are quite similar. At room temperature the principal absorptions occur at the same peak position, 680 mμ, and have nearly the same half-width of about 0.36 ev. At liquid helium temperature corresponding structure in both crystals appears at approximately 387, 490, 580, 620, and 640 mμ. Figure 2 shows that the main components in the additively colored crystal can be resolved[5] into bands which peak at 616 and 642 mμ. A detailed comparison of the x-rayed and additively colored crystals indicates that this major structure is present in substantially the same relative proportion in both crystals. No particular significance is attached to the fact that the 642-mμ band in the additively colored crystal (Fig. 2) is slightly displaced from the 640-mμ absorption in the x-rayed crystal (Fig. 1); a small underlying absorption could

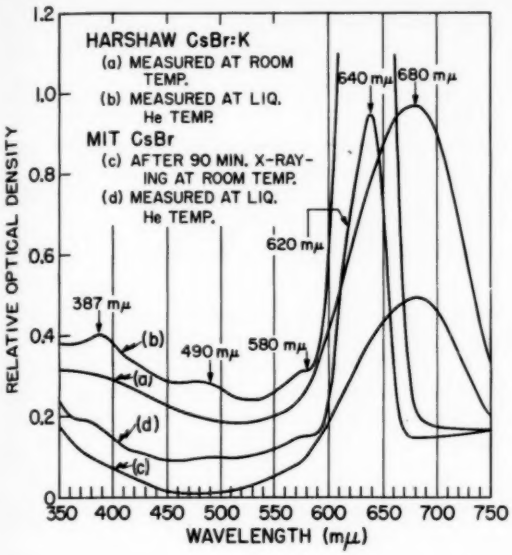

FIG. 1. The optical absorption spectra of Harshaw CsBr, additively colored with potassium, at room temperature (a) and liquid helium temperature (b), and MIT-grown CsBr x-rayed for 90 minutes at room temperature, measured optically at room temperature (c) and liquid helium temperature (d).

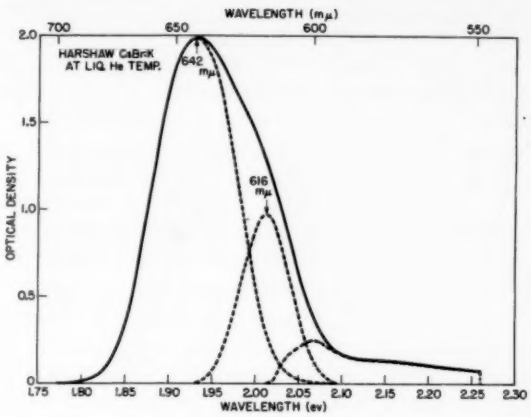

FIG. 2. The major absorption structure of additively colored Harshaw CsBr at liquid helium temperature (solid line) and its resolved substructure (dashed line).

easily account for this difference. For a similar reason, the relative proportion of the weak absorptions at shorter wavelengths in Fig. 1 has not been analyzed in detail, although it appears that these bands are present in roughly the same relative proportion to the major components. It has also been observed that the 580-, 620-, and 640-mμ bands occur in both Harshaw and MIT-grown CsBr that have been x-rayed directly at liquid helium temperature, and again they appear in the same relative proportions as the data of Figs. 1 and 2, even after successive optical bleachings. The presence of short-wavelength structure corresponding to the 387- and 490-mμ bands has not been verified after liquid helium temperature x-raying due to the introduction of other overlapping bands by the low-temperature irradiation.

From what is known of the behavior of the F center in NaCl-type alkali halides the identification of the room-temperature 680-mμ absorption band in CsBr as the F band is almost certainly correct.[6,7] It is the principal optical absorption; it appears to be a simple band; it bleaches under visible light; it is identical in both the x-rayed and additively colored specimens; and its peak position obeys a Mollwo relation[6] with what appears to be the F band in cesium halides of the CsCl structure. Furthermore, as the temperature is lowered a progressive shift to shorter wavelengths and a sharpening is observed.[8] The fact that the band does not remain simple but shows substructure at liquid helium temperature is contrary to the normal behavior of the F band in NaCl-like alkali halides.[9] The evidence, however, for believing that this structure (or at least a portion of it) is related to the F center is the following: First, the main components are always present in definite proportion independent of the manner in which the absorption is produced. This would be unlikely if only one of the components was the F band and all the others were unrelated to the F band. Second, the same structure appears in both x-rayed and additively colored specimens. This situation is again most easily understood if a single excess-electron center like the F center is involved.[10] The possibility of associating the band structure with other centers, like the M and R centers, is contradictory to the known relationship of these centers to the F center in the NaCl-type systems. In general they appear as less intense bands on the long-wavelength side of the F band, they are not necessarily in fixed proportion relative to one another, and they are not formed on irradiating at liquid helium temperature.

The possibility that these sub-bands represent different excited states of the F center imme-

diately suggests itself. The general decreasing strength of the first three major components in moving to shorter wavelengths is in accord with this suggestion,[11] despite the fact that the oscillator strength[12] of the main band is considerably lower than values found for the oscillator strength of the F center in NaCl-type alkali halides. Additional measurements, particularly measurements of spin resonance and the temperature dependence of photoconductivity, could check on the possibility of multiple excited F-center states and give further insight into the nature of these states.

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[1]See F. Seitz, Revs. Modern Phys. 26, 7 (1954).

[2]H. Rabin, J. H. Schulman, and R. F. Marzke, Bull. Am. Phys. Soc. 5, 48 (1960).

[3]This crystal was kindly provided by Dr. W. G. Maisch of the University of Maryland.

[4]Kindly provided by Professor A. Smakula and Dr. P. Avakian at the Massachusetts Institute of Technology who are also currently investigating color center formation in the cesium halides.

[5]This resolution is performed by assuming the chief band peaks at 642 mμ, shorter wavelength bands do not extend to wavelengths longer than 642 mμ, and the band is symmetrical on an energy scale. Similarly, it is assumed the second band which peaks at 616 mμ is not overlapped to the long-wavelength side of 616 mμ by shorter wavelength structure, namely the 580-mμ band, and it is also symmetrical. It is clear from Fig. 2 that the structure remaining does not give a simple symmetrical band at 580 mμ. However, it is to be remembered that the background absorption and errors in separating the components are lumped in this short-wavelength remainder.

[6]P. Avakian and A. Smakula, Bull. Am. Phys. Soc. 5, 48 (1960).

[7]C. J. Rauch and C. V. Heer, Phys. Rev. 105, 914 (1957).

[8]At liquid nitrogen temperature there appears to be an increased distortion in the general structure of the band, but no clearly resolved substructure appears as it does at liquid helium temperature.

[9]See, for example, W. H. Duerig and J. J. Markham, Phys. Rev. 88, 1043 (1952).

[10]The possible role of impurity atoms can not be totally ignored. H. W. Etzel and J. H. Schulman, J. Chem. Phys. 22, 1549 (1954), have shown that the same optical impurity bands produced in x-rayed NaCl doped with silver can be produced by electrolytic coloration. It is well known, of course, that the additive coloration process is quite analogous to that of electrolytic coloration. Since both Harshaw and MIT-grown CsBr crystals have been used in the present study, their impurity content would not necessarily be expected to be identical.

[11]F. Lüty has recently discussed possible excited F-band states in NaCl-type alkali halides in which the relative magnitudes of the bands are not of successively decreasing absorption strengths in passing to shorter wavelengths. This work was presented at the International Symposium on Color Centers in Alkali Halides, Oregon State College, September, 1959 (unpublished).

[12]Rauch and Heer[7] give the F-center oscillator strength as 0.26 assuming a single Gaussian band at room temperature. Since the 642-mμ band constitutes about 69% of the total area of the absorption structure of Fig. 2, the oscillator strength of the first excited state would be 0.18; similarly the second excited state (616-mμ band) would be about 0.05.

SEMICONDUCTING PROPERTIES OF CUBIC BORON PHOSPHIDE

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(Received February 15, 1960)

In recent years there has been considerable interest in group III-group V compound semiconductors as well as large band gap semiconductors. Cubic boron phosphide is such a material. It has been prepared and identified previously[1] and the present paper describes preliminary measurements of its semiconducting properties.

Cubic BP has been prepared by two basically different methods: (1) by the vapor phase reaction of boron compounds with phosphorus compounds or phosphorus vapors; and (2) by the slow cooling of a melt containing dissolved boron and phosphorus.

The BP prepared by the vapor phase reactions[2] has very small crystal size and varies from reddish tan to black in color. However, this material could be deposited in the form of films which were useful for optical absorption measurements since they could be made very thin. A variation of the vapor phase reaction resulted in small "needle" or "blade" shaped crystals. These crystals are 1 mm or 2 mm in length and reddish brown in color and transparent.

The method involving slow cooling of a melt results in crystals which are on the order of

1 mm in all dimensions and of a similar color. These crystals are also transparent and occasionally a crystal can be found with a pair of flat, parallel faces. Measurements of the displacement of an image by such a crystal give a rough value of index of refraction of between 3.0 and 3.5.

The x-ray powder diffraction pattern of these materials is identical to that reported for cubic BP by other workers.[1] Elemental analyses give boron to phosphorus ratios of unity within about 1% which is the accuracy of the method.

BP is quite inert chemically. It is not attacked by mineral acids or any of the usual semiconductor etches. It is attacked slightly by molten NaOH. BP has a hardness about equal to that of silicon carbide (Knoop hardness number of 3200 for 100-g load).

Ruehrwein and Williams[3] of this laboratory have found by means of optical absorption measurements on thin polycrystalline films, that the energy gap of BP is almost 6 ev. Figure 1(a) shows representative measurements[3] on films of different thicknesses. If one neglects scattering effects, the absorption coefficient at 0.3 micron is about 5×10^4 cm^{-1}. These films were deposited on quartz plates. The absorption of the quartz is negligible in this region. The value of energy gap of about 6 ev is consistent with what one would expect on the basis of lattice parameter considerations. Note the shoulder around 0.6 micron accounting for the reddish color which this material exhibits by transmission.

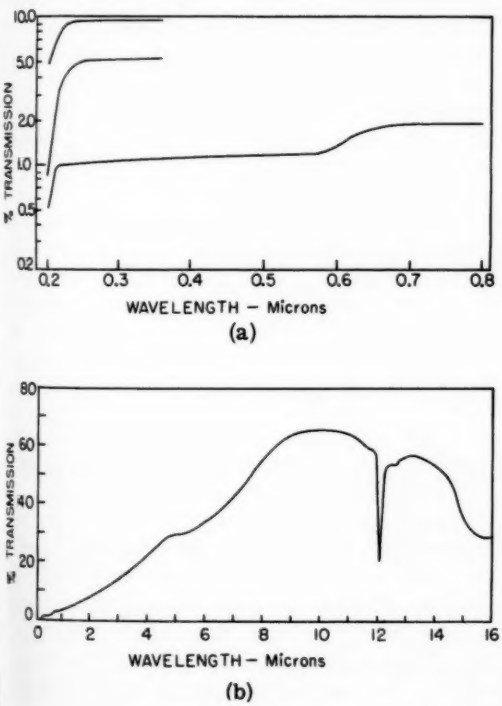

FIG. 1. (a) and (b) Percent transmission of polycrystalline films of boron phosphide.

Absorption measurements on a polycrystalline film in longer wavelength regions are shown in Fig. 1(b). For this measurement the film was stripped off its quartz backing. The gradual increase in transmission out to about 10 microns is probably due to a decrease in scattering at longer wavelengths. The sharp absorption at 12.1 microns is presumably a lattice absorption.

Optical absorption measurements of single-crystal samples are more difficult because of the small size of the crystals available and the difficulty in producing thin samples. The sensitivity is too poor to show the fundamental absorption and the only feature observed is the absorption responsible for the color of the crystals. The highest absorption coefficient measured on a single-crystal sample is 10^3 cm^{-1}.

Preliminary Hall measurements have been made on crystals grown by method 2. The sample geometry was determined by their natural crystal faces. Electrical contact was made using silver paint, tungsten, or platinum points. The lead resistance could usually be decreased considerably by the use of a capacitor discharge. The measurements show that these crystals are p-type with a carrier concentration ranging from 1 to 5×10^{18} carriers/cc. The Hall coefficient is essentially constant from about 900°K down to about 160°K. Difficulties with leads prevented Hall measurements at lower temperatures. The shape of the resistivity curve at low temperatures suggests that the material is still in the exhaustion range at 78°K. Figure 2 gives Hall mobility on such p-type BP as a function of $1000/T$. At room temperature the mobility is decreasing with decreasing temperature, characteristic of impurity scattering. Thus the room temperature lattice mobility of p-type BP must be considerably greater than the measured value, perhaps as large as 300 or 500 cm^2/volt second.

Thermoelectric power and rectification measurements on the needle-shaped crystals indicate that these crystals are n-type. Crude resistivity measurements indicate that the impurity level may be around 10^{17} carriers/cc if we assume the electron mobility to be of the same order of

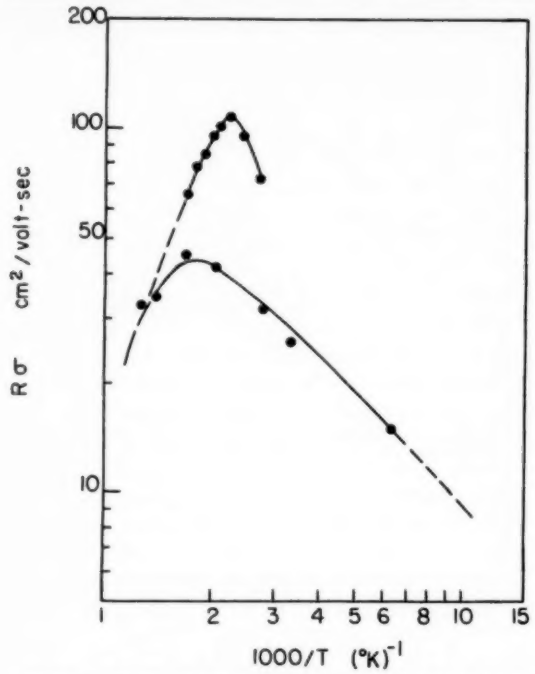

FIG. 2. Hall mobility of single-crystal p-type boron phosphide.

magnitude as the hole mobility.

The thermoelectric power of both the n- and p-type material is about 300 microvolts/°C. Point contact rectification has been observed with both n-type and p-type BP. Rectification ratios of greater than 10^3 to 1 have been observed at both room temperature and 400°C.

It is necessary to comment concerning the color of the BP obtained to date. A semiconductor with an energy gap of nearly 6 ev should be transparent and colorless, while the BP obtained has a reddish color. The optical absorption responsible for this color occurs at a wavelength corresponding to an energy of about 2 ev. Such a level would not be ionized at the temperature of the measurements, so it would not contribute carriers, even if this were possible. There are several possible explanations for this absorption but more work will be necessary before any conclusions can be reached.

In summary it has been shown that it is possible to produce single crystals of cubic BP of measurable size, and although the material prepared so far is impure, by semiconductor standards, and although the measurements just presented are preliminary in nature, the properties of BP appear to be interesting and promising.

[1]P. Popper and T. A. Ingles, Nature 179, 1075 (1957).

[2]F. V. Williams, paper presented at American Chemical Society Meeting, Boston, Massachusetts, April, 1959 (unpublished).

[3]R. A. Ruehrwein and F. V. Williams (unpublished data).

MECHANISMS OF VOLUME SELF-DIFFUSION IN α-Fe AND γ-Fe†

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(Received January 11, 1960)

Several years ago, Kuczynski[1] introduced a method for determining self-diffusion coefficients without the aid of radioactive tracers. An expression was derived which relates the dimensions of a sintered interface between two particles to D, the coefficient of volume self-diffusion. Although this method will never supplant the tracer method for determining diffusion parameters, the importance of its use should not be overlooked in diffusion studies (*a*) where no suitable isotopes are available and (*b*) where it is desired to distinguish various diffusion phenomena which have different causes. As long as 50 to 100 measurements of separate fillets are made for each point on the $\ln D$ vs $1/T$ curve, the statistics will be favorable to enable the determination of D within a factor of 2 or 3 with about an 80% confidence limit for most metals.

An adaptation of this method has been used to determine the self-diffusion parameters in both face-centered cubic (fcc) and body-centered cubic (bcc) thorium,[2] where no suitable isotope for tracer studies is available. In this latter work, it was found that near the transition temperature, D in bcc thorium is approximately a factor of five less than D in fcc thorium. In contrast, radioactive tracer studies[3,4] show that D in bcc iron is approximately 400 times larger

than D in fcc iron, near the transition temperature. It was therefore suggested[2] that sintering studies be conducted on iron in an attempt to determine the cause of this extremely large difference in D for the fcc and bcc structures.

It is important to note that the volume diffusion mechanism responsible for sintering phenomena must involve lattice defects, e.g., interstitials or vacancies, since a net transport of matter must take place to establish the sintered fillet. On the other hand, the total diffusion which is observed through use of radioactive tracers may have occurred either by such a defect mechanism or by a direct interchange or ring mechanism,[5] or possibly by a combination of both kinds of mechanisms. The direct interchange mechanism cannot contribute to the formation or growth of the sintered fillet since it does not result in any net mass transport. Consequently, a comparison of diffusion data obtained from tracer and sintering studies should allow one to determine the relative contributions of those diffusion mechanisms which can produce net mass transport and those mechanisms which cannot.

In the present study, 0.020-inch diameter wires were fabricated from Johnson-Matthey (99.99% pure) iron. These wires were wrapped in four spools, each containing about four layers, in such a way that approximately one hundred possible sintered interfaces were available for examination for each cross-sectional cut. These spools were pre-sintered in hydrogen at 950°C for one hour. Two of the spools were sectioned and measurements were made on the sintered interfaces. These results established the dimensions of the fillets at "zero" time; this procedure also eliminates errors due to distorting the interface region during wrapping of the spools. One of the sectioned spools and one of the whole spools were then heated in vacuum ($<10^{-5}$ mm Hg) for 100 hours at 935°C; the other spools were treated similarly at 885°C. Following these treatments, final measurements of the sintered interfaces were made and the growth of each fillet in this time period was determined by subtracting the initial from the final values. Approximately 100 separate fillets were measured for each temperature and these values were averaged to obtain the value from which D was determined. The previously derived formula[2] relating D to the fillet measurements is

$$D = ka^3T(2\delta^3t\gamma)^{-1}y^5f(y),$$

where k = Boltzmann constant, a = wire radius, δ = interatomic distance, T = absolute temperature, t = time, γ = surface energy, y = ratio of fillet width and wire diameter, and $f(y) = 0.0637 + 0.1096y + 0.1426y^2 + 0.1668y^3 + \ldots$

For $\gamma = 2000$ dynes/cm, the values of D determined from this expression for the two temperatures are:

$$D_{935} = 3.5\times10^{-13}\ \text{cm}^2/\text{sec (fcc)},$$

$$D_{885} < 6\times10^{-14}\ \text{cm}^2/\text{sec (bcc)}.$$

Although some sintering did occur at 885°C, the diffusion rate was so slow that after the 100-hour treatment, it was only possible to place an upper limit on D_{885}. These values are plotted in Fig. 1 along with the previously mentioned data obtained using radioactive tracers. More recent tracer data[6] on α-iron are also shown. Approximately 85% of the present individual measurements of D_{935} fall within the error bar shown in Fig. 1. The sintered fillet is a result of contributions

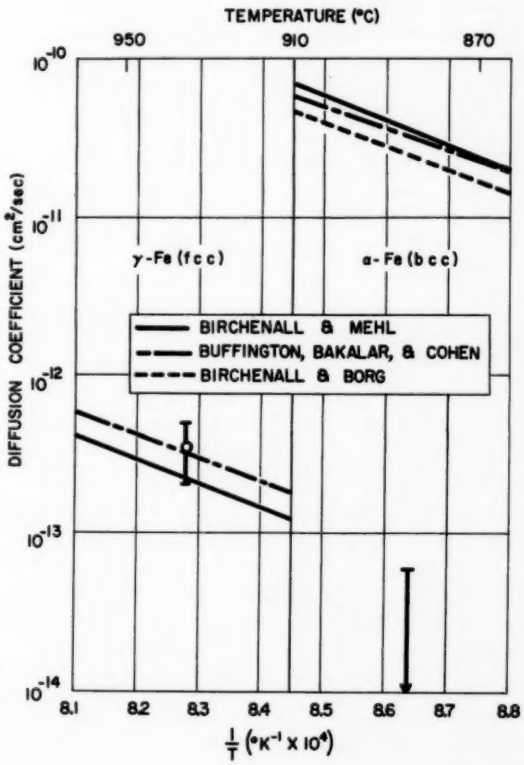

FIG. 1. Plot showing comparison of $\ln D$ vs $1/T$ for α-Fe and γ-Fe as determined from radioactive tracer data and the present sintering data (Φ, $\downarrow$).

from several diffusion mechanisms which employ a defect mechanism of mass transport. If volume diffusion is not predominant in establishing the fillet, then the value of D_{885} corresponding to volume diffusion must be even smaller than that given in the paper, thus increasing the discrepancy between D_{885} determined from sintering data and the corresponding value determined from tracer data. This strengthens the main argument, namely, that different mechanisms of diffusion are dominant in α-Fe and γ-Fe.

From the preceding discussion and the fact that D_{935} from sintering data is in good agreement with the tracer data, it is concluded that (*a*) the sintered fillets in the fcc structure were established by volume self-diffusion via a defect mechanism, presumably vacancies, and (*b*) a diffusion mechanism which does not involve lattice defects is primarily responsible for volume diffusion in bcc iron. This latter mechanism is presumed to be a ring mechanism such as proposed by Zener, where two or more neighboring atoms exchange positions simultaneously. The contribution from a four-atom ring seems most favorable from theoretical energy considerations.[7] Therefore, it is concluded that the large change in D, which occurs when iron is heated through the alpha-gamma critical temperature, is caused by a change in the dominant diffusion mechanism. It seems most likely that this change is from a four-atom ring mechanism in bcc iron to a vacancy mechanism in fcc iron. This conclusion is further substantiated by recent diffusion studies on α-iron in a high-temperature gradient,[8] and is direct experimental support for the theoretical treatment of Le Claire[9] on diffusion mechanisms in bcc metals.

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†This work was performed under contract with the U. S. Atomic Energy Commission.

[1]G. C. Kuczynski, J. Appl. Phys. 21, 632 (1950).
[2]C. J. Meechan, Advances in Nuclear Engineering, edited by J. R. Dunning and B. R. Prentice (Pergamon Press, New York, 1957), p. 209.
[3]C. E. Birchenall and R. F. Mehl, Trans. Am. Inst. Mining Met. Petrol. Engrs. 188, 144 (1950).
[4]F. S. Buffington, I. D. Bakalar, and M. Cohen, Trans. Am. Inst. Mining Met. Petrol. Engrs. 188, 1375 (1950).
[5]C. Zener, Acta Cryst. 3, 346 (1950).
[6]C. E. Birchenall and R. J. Borg (private communication).
[7]C. Zener, Imperfections in Nearly Perfect Crystals, edited by W. Shockley (John Wiley & Sons, Inc., New York, 1952), p. 289.
[8]W. G. Brammer, Acta Met. (to be published).
[9]A. D. Le Claire, Acta Met. 1, 438 (1953).

SHELL EFFECTS IN THE OPTICAL POTENTIAL

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(Received December 7, 1959)

There are a few qualitative disagreements between the observed low-energy neutron scattering data (the strength function $\langle\Gamma^0/D\rangle$ and the scattering length R') and the predictions from the conventional optical potential whose imaginary part (W) is independent of the mass number A. (1) The observed $\langle\Gamma^0/D\rangle$ is too small around $A\sim100$,[1] that is, W should be smaller than usual in this region. (2) The observed R' is too small around $A\sim60$,[2] (Fig. 1), which means that W should be larger than usual in this region. (3) The observed $\langle\Gamma^0/D\rangle$ is too small around $A\sim60$,[2] (Fig. 1), and this also indicates that W is larger than usual here.

Lane et al.[1] attributed the phenomenon (1) to the fact that for $A\sim100$ either proton or neutron is near magic. This will not be the principal reason because for $A\sim40$, where both proton and neutron are magic, the experiments agree fairly well with the theory (Fig. 1).[3] Phenomena (2) and (3) were attributed to the quadrupole deformation of the nucleus by Seth,[4] as was done for $A\sim150$. This again is doubtful since, for $A\sim60$, the quadrupole moments are not too large to be explained by configuration mixing and the low-lying levels are not of rotational type.

We propose to explain the above phenomena by the model of Lane and Wandel.[5] Usually this model is applied to an infinite medium. Now we apply it to a finite nucleus. The theoretical justification for this was given before.[6] What is the difference between the two? The difference appears in the conservation law for the scattering process leading to the absorption. For an

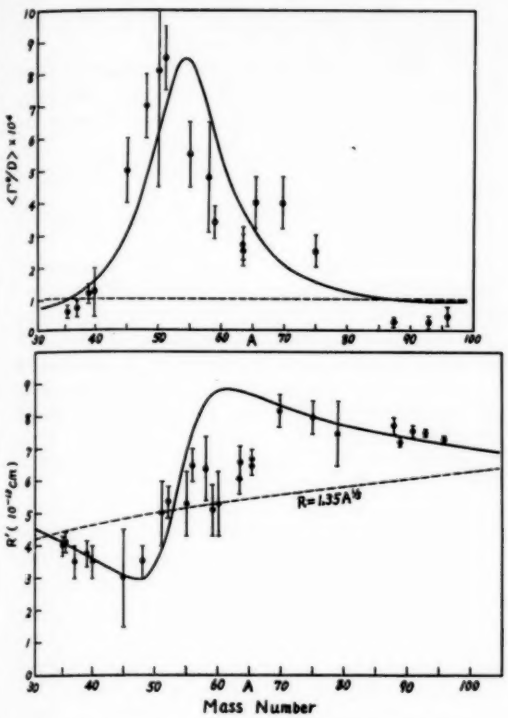

FIG. 1. The observed strength function $\langle \Gamma^0/D \rangle$ and the scattering length R'.[2] The solid curve is the prediction from the conventional optical potential [V. F. Weisskopf, Physica 18, 952 (1956)]. The dashed curve is the one from the black nucleus.

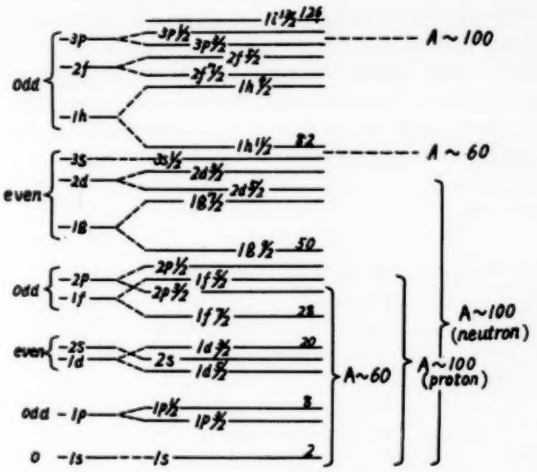

FIG. 2. The schematic single-particle levels.[7] The filled levels are indicated by curly brackets, and the positions of the zero-energy incident neutron are indicated by dashed lines for $A \sim 60$ and $A \sim 100$. For the latter, the top of the filled levels is different for protons and neutrons.

infinite nucleus, we calculate the effective two-body scattering cross section in the nuclear matter for which the momentum and energy are conserved. For a finite system, on the other hand, it is the angular momentum, parity, and energy which are to be conserved. The parity conservation is very important since, according to the shell model, the single-particle levels with the same parity are grouped together[7] (except for one level with the highest angular momentum) and the conservation law is either yes or no. The conservation of the angular momentum is not so restrictive because of its vectorial nature.

Figure 2 represents the schematic single-particle levels.[7] In the figure the filled levels are indicated by curly brackets and the positions of the zero-energy s-wave neutron are indicated by dashed lines for $A \sim 60$ and 100. For $A \sim 60$: (i) The incident s neutron can easily jump down to the $3s$ state just below and raise a nucleon in the target to a nearby state. (ii) If the incident neutron jumps down far below and the parity change is yes, a nucleon in the target is raised high so that the energy is conserved and the parity change is again yes. So we expect large W. For $A \sim 100$: (i) When the incident s neutron jumps down to the nearby states, the parity change is yes, but for the raised nucleon in the target the parity change is essentially no. (ii) If the incident neutron jumps down far below, the nucleon in the target is raised high and the parity law is again not satisfied. Thus we expect small W. In this way we can understand (1), (2), and (3) as the shell effect, but the meaning is quite different from that of Lane et al. The sharp rise of $\langle \Gamma^0/D \rangle$ and R' near $A = 70$ (Fig. 1) may be due to the filling of the minus parity levels.[8]

According to the present theory, W for the p state must be small for $A \sim 60$ and large for $A \sim 100$. There are no reliable data for W in the p state. Seth,[4] and also Lane et al.,[1] gave some experimental data indicating that W is small for $A \sim 100$ in contradiction to our theory, but these are not convincing. An exact analysis for W in the p state is highly desirable.

The surface absorption model[9] also makes $\langle \Gamma^0/D \rangle$ small for $A \sim 100$ but, according to our

preliminary machine calculations, the reduction is not enough. Further, this model does not explain (2). The nonlocal nature[6] of the imaginary part may also be responsible for making the effective local W dependent on A and the incident angular momentum. This point has not been studied yet.

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[1]A. M. Lane, J. E. Lynn, E. Melkonian, and E. R. Rae, Phys. Rev. Letters 2, 424 (1959).

[2]K. K. Seth, Revs. Modern Phys. 30, 442 (1958). Figure 1 is reproduced from a private communication from Seth.

[3]A. M. Lane et al. stated that W is small since the level density is small near magic. W depends, however, also on the nuclear matrix elements which will be large if the level density is small. Further, we do not observe large difference between $\langle \Gamma^0/D \rangle$ for the even nucleus and the odd nucleus although the level density differs by about a factor 2. This was pointed out to the author by V. F. Weisskopf.

[4]K. K. Seth, Proceedings of the International Conference on the Nuclear Optical Model, Florida State University Studies, No. 32 (The Florida State University, Florida, 1959).

[5]A. M. Lane and C. F. Wandel, Phys. Rev. 98, 1524 (1955).

[6]A. Sugie, Progr. Theoret. Phys. (Kyoto) 21, 681 (1959).

[7]M. G. Mayer and J. H. D. Jensen, Elementary Theory of Nuclear Shell Structure (John Wiley and Sons, Inc., New York, 1955), p. 50.

[8]From Fig. 1 we see a small discrepancy in $\langle \Gamma^0/D \rangle$ near $A \sim 50$ but, since R' is normal here, the theoretical R' and $\langle \Gamma^0/D \rangle$ will be fitted to the experiment by slightly changing the depth (or the radius) of the real part of the potential, leaving W near the normal value.

[9]F. Bjorklund and S. Fernbach, Phys. Rev. 109, 1295 (1958).

FURTHER INTERPRETATION OF A MEDIUM ENERGY $(p, 2p)$ EXPERIMENT*

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(Received February 19, 1960)

In a previous publication,[1] the momentum transfer distribution observed in the 40-Mev $(p, 2p)$ angular correlation experiment of Griffiths and Eisberg[2] was interpreted as giving information about the momentum distribution of nucleons localized in the nuclear surface. After unfolding the momentum transfer distribution due to the optical potentials in the entrance and exit channels, it was found that momentum components corresponding to proton energies of about 1 Mev were most likely to be observed for the bound particle. It is the purpose of this Letter to point out some additional considerations which contribute to an understanding of the reaction.

A collision of a nucleon with a stationary free nucleon will result in an angle of 90° between the momenta of the particles in the final state. In reference 1, it is shown that the effect of the binding potential is to move the region of smallest momentum transfers to an angle less than 90°. If small momentum transfers are most probable, the result will be an angular correlation peaked at an angle less than 90° (depending on the binding and excitation energy) as is observed experimentally.

In reference 1, it was assumed that the momentum distribution was isotropic. The momentum components were estimated from the experiment. Essentially the peak in the experimental angular correlation is so narrow that only small momenta (~1 Mev) contribute. The relationship between the width of the angular correlation curve and the momentum of the struck particle is explained in reference 1.

It was also shown in reference 1 that the reactions are not isotropically distributed over the nuclear surface. They come predominantly from an equatorial belt on the surface with the incident direction as the north-south axis. Within the belt, the main contribution comes from regions A, Fig. 1, since, for one particle scattered to the left and the other to the right, one of the particles coming from region B is likely to be

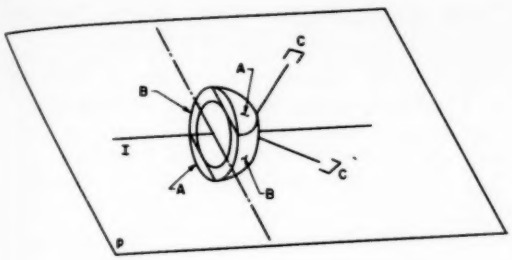

FIG. 1. Schematic diagram of a nucleus and experimental arrangement showing: P, a plane passing through the counters and the incident beam that also passes through the center of a nucleus; C, counters; I, incident beam; A and B, regions of greater and lesser importance in the $(p,2p)$ reaction, respectively.

absorbed. From simple kinematics with particles of equal mass, it may be shown that if the incident particle collides with a free particle moving at 90° with respect to it in the laboratory system, the angular correlation between the particles after collision is still given by a δ function at 90° in the laboratory system. In the approximation that the collision occurs at the equator, only the tangential component of the momentum of the struck particle in the incident direction will contribute to a spreading of the angular correlation. From the consideration of proton flux in an optical potential[3] it is observed that the incident protons travel roughly tangentially in the region of interaction.

Reactions are most likely to be observed in which the struck particle is moving tangentially. Radial motion will result in the plane of the collision either intersecting the nucleus in which case at least one particle is likely to be absorbed (regions B, Fig. 1) or out of the plane of the counters (region A, Fig. 1).

It has been shown by Baker, McCarthy, and Porter[4] that the shape of the momentum distribution that may be observed for particles localized at the nuclear surface is determined almost completely by the localization rather than by the wave function of the bound particles. The tangential momentum components for particles in higher shell model states are found to be much less than the radial components, within the restrictions imposed by the localization. The results of this calculation agree with the uncertainty principle because the region of interaction is less restricted by absorption tangentially than radially. Thus a more realistic theoretical estimate of the effective momentum distribution than the isotropic assumption of reference 1 is possible. Using the above momentum transfer components in the relationship between momentum transfer and angular correlation (Figs. 2 and 3 of reference 1), the experimental angular correlation curve may be easily understood both in position and width.

Owing to the imperfection of the tangential approximation, there will be a contribution from the tangential component perpendicular to the incident beam and from the radial component. Although they cannot appreciably spread the angular correlation, they will cause a preponderance of events in which the energies of the final particles differ if the counters are placed fairly symmetrically about the incident direction [see Fig. 2]. This effect was observed in the experiment.

The small momentum transfers observed in the $(p,2p)$ experiments are essentially due to the anisotropy of the momentum distribution and the fact that the interaction does not effectively occur uniformly throughout the nuclear surface. Thus

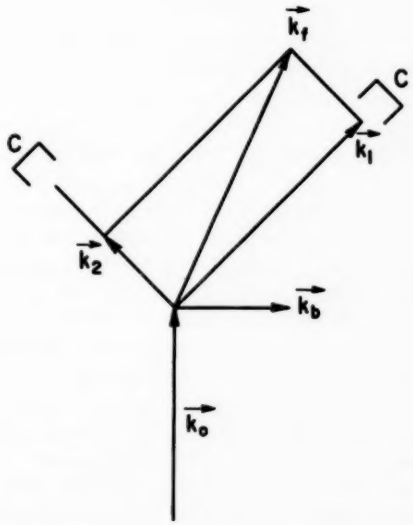

FIG. 2. Sketch showing conservation of momentum under certain conditions: $\vec{k}_0$ = incident momentum; $\vec{k}_b$ = radial component of the momentum of a bound particle (region B) = the tangential component of the momentum of a bound particle that is perpendicular to the incident momentum (region A); $\vec{k}_f$ = final momentum; $\vec{k}_1$, $\vec{k}_2$ = momenta of particles in the final state. Conservation of momentum requires $k_1^2 > k_2^2$.

one would suspect that a theory analogous to the Butler theory should not predict the correct, rather narrow angular correlation since (*a*) it does not contain information about the momentum distribution in different regions of the nuclear surface, and (*b*) it does not distinguish between the regions of the nuclear surface as to which are more likely to contribute to the reaction. Calculations using such a theory confirm this prediction.

*Supported in part by the National Science Foundation and the U. S. Atomic Energy Commission.

[1]I. E. McCarthy, E. V. Jezak, and A. J. Kromminga, Nuclear Phys. 12, 274 (1959).

[2]R. J. Griffiths and R. M. Eisberg, Nuclear Phys. 12, 225 (1959).

[3]I. E. McCarthy, Nuclear Phys. 10, 583 (1959).

[4]G. A. Baker, I. E. McCarthy, and C. E. Porter (to be published).

NUCLEAR MATRIX ELEMENTS IN THE BETA DECAY OF Sb^{124}†

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After the recent clarification of the beta interaction it has become of interest to study the relative contributions of the various matrix elements to first forbidden β transitions. It is the purpose of this paper to demonstrate that an unambiguous determination of matrix elements in a nonunique β transition (e.g., Sb^{124}) is possible on the basis of precise β-γ directional and β-γ circular polarization correlation measurements, if the β transition shows appreciable deviation from the ξ approximation.[1-3]

The ξ approximation was first introduced by Konopinski and Uhlenbeck[1] to explain the statistical shape of most nonunique first-forbidden beta spectra. In this approximation the beta transition probability is expanded in powers of the nuclear radius R and only the leading terms are taken into account, which are associated with the Coulomb factor $\xi = \alpha Z/2R$. Deviations from this approximation may be caused by selection rule effects which inhibit contributions from matrix elements other than $\int B_{ij}$. The contribution of the $\int B_{ij}$ term, which is of rank $\lambda = 2$ and which describes the component of the lepton field carrying away two units of angular momentum, may then become very important. The spectra of such β transitions exhibit deviations from the statistical shape and their ft values ($\log ft > 10$) are considerably larger than the characteristic ft values of nonunique first forbidden transitions ($\log ft \cong 8$).

It was suggested[2,3] that such a selection rule effect rather than a mutual cancellation of matrix elements explains the large ft value of the 2.31-Mev β transition of Sb^{124} ($\log ft = 10.6$). The results of the present investigation confirm this hypothesis.

The angular and energy dependence of the β_1-γ_1 directional correlation of Sb^{124} involving the β_1 component of 2.31-Mev maximum energy (refer to inset of Fig. 1) was measured with the vacuum chamber described previously.[4] The directional correlation $W_{\beta\gamma}(\theta, \overline{W}_\beta = 4.8)$ of the β_1-γ_1 cascade measured at a fixed average energy of $\overline{W}_\beta = 4.8$ (in units of mc^2) is shown in Fig. 1. A least-squares fit of the experimental points to the correlation function:

$$W_{\beta\gamma}(\theta, \overline{W} = 4.8) = 1 + A_2(4.8)P_2(\cos\theta) + A_4(4.8)P_4(\cos\theta), \quad (1)$$

yielded the following values for the correlation coefficients:

$$A_2(4.8) = -0.390 \pm 0.011,$$
$$A_4(4.8) = +0.004 \pm 0.013. \quad (2)$$

The absence of a $P_4(\cos\theta)$ term provides further evidence against the decay scheme $4^+(\beta_1)2^+(\gamma_1)0^+$.

The dependence of the coefficient $A_2(W)$ on the β energy is shown in Fig. 2. A simultaneous measurement of the energy dependence of the β_2-γ_2 directional correlation made it possible to correct the data for the presence of the β_2-γ_1 directional correlation at β energies below the maximum energy of the β_2 spectrum ($W_0 = 4.15$). There is, however, some uncertainty in this correction due to the fact that the sign of the $E2$-$M1$ mixing ratio δ of the γ_2 transition is not

known ($\delta = \pm 1.00 \pm 0.085$).[5] The error caused by this uncertainty is included in the error flags of the experimental points corresponding to $W < 4.15$.

All available data indicate strongly that the β_1-γ_1 cascade of Sb^{124} follows the decay scheme $3^-(\beta_1)2^+(\gamma_1)0^+$. The four matrix elements which can contribute to a first forbidden β transition with $\Delta I = \pm 1$, yes, are the relativistic matrix element $y = C_V \int i\vec{\alpha}$, the moment type matrix elements $x = -C_V \int \vec{r}$, and $u = C_A \int i\vec{\sigma} \times \vec{r}$ (all of rank $\lambda = 1$) and the matrix element $z = C_A \int B_{ij}$ ($\lambda = 2$).

After Kotani[2] the energy dependence of the directional correlation coefficient $A_2(W)$ is expressed by

$$A_2(W) = \frac{W^2-1}{W} \frac{R_3 + eW}{1 + aW + cW^2 + (b/W)}. \quad (3)$$

The coefficients R_3, e, a, b, and c, which are complicated functions of the matrix element parameters x, u, z, and $Y = y - \xi(u+x)$ and of the maximum energy W_0 are given in reference 2.

By a least-squares method the values of the

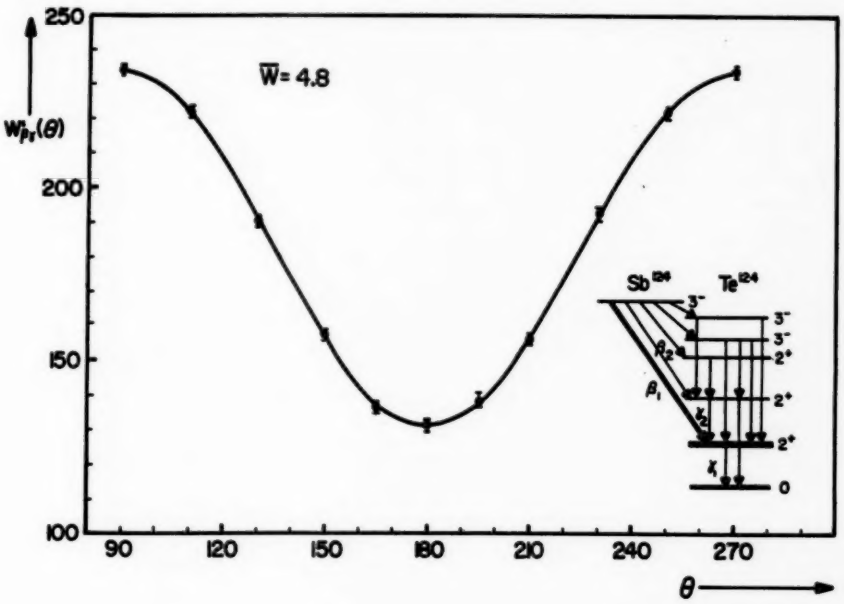

FIG. 1. Angular dependence of the β-γ directional correlation involving the 2.31-Mev β transition of Sb^{124}. The measurements were made at an average β energy of $\bar{W} = 4.8$ (in units mc^2).

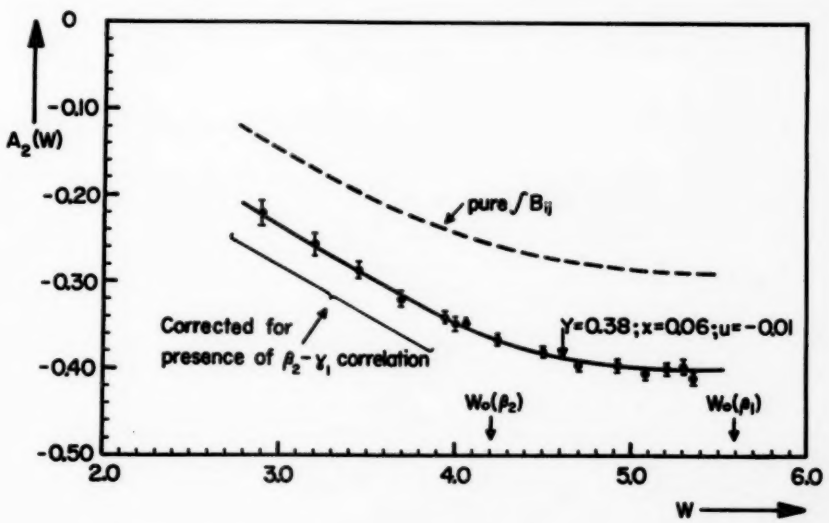

FIG. 2. Energy dependence of the anisotropy factor $A_2(W)$ of the β_1-γ_1 directional correlation. The solid line represents $A_2(W)$ calculated with the parameters $u = -0.01$, $x = 0.08$, $Y = 0.38$. The dashed line corresponds to a pure $\int B_{ij}$ (unique) transition.

parameter ratios $u/z, x/z$, and Y/z were determined which resulted in a best fit of the data of Fig. 2 to a curve given by Eq. (3). As an additional condition it was imposed that the set of parameters also satisfy the circular polarization correlation measurements of Hartwig and Schopper,[6] i.e., $P_C(126°)/P_C(160°) = -2$, where $P_C(\theta)$ is the degree of circular polarization of the γ_1 radiation measured at the angle θ. The fit of the data yields for the nuclear parameters:

$$u = -(0.01 \pm 0.04)z,$$
$$x = (0.08 \pm 0.08)z,$$
$$Y = (0.38 \pm 0.12)z. \qquad (4)$$

This set of parameters agrees satisfactorily with the set obtained by Hartwig and Schopper,[6] which was determined on the basis of a somewhat different approach. The function $A_2(W)$ calculated with the parameters of Eqs. (4) is represented as solid line in Fig. 2. For comparison the curve $A_2(W)$ corresponding to a unique β transition (pure $\int B_{ij}$) is included.

By taking into account the corrected ft value of the 2.31-Mev β transition, $ft = 10^{10.6}$ sec, or $ft = 3.1 \times 10^{31}$ in units $\hbar = m = c = 1$, the absolute values of the matrix elements involved in this β decay can be computed[7]:

$$|\int B_{ij}|/R = (1.20 \pm 0.15) \times 10^{-2},$$
$$|\int \vec{r}|/R = (1.2 \pm 1.2) \times 10^{-3},$$
$$|\int i\vec{\sigma} \times \vec{r}|/R = (0.1 \pm 0.4) \times 10^{-3},$$
$$|\int i\vec{\alpha}| = (3.1 \pm 2.4) \times 10^{-4},$$
$$(\int i\vec{\alpha} / \int B_{ij}) > 0. \qquad (5)$$

The values of the matrix elements are given in a form which is independent of the chosen system of units (R = nuclear radius). In addition, the lack of overlap of the nuclear wave functions which occur in the matrix elements is more evident in the form of Eqs. (5). If the wave functions of the initial and final nuclear states would overlap perfectly, the values of $\int B_{ij}/R$, $\int \vec{r}/R$, and $\int i\vec{\sigma} \times \vec{r}/R$ would be of order unity whereas the relativistic matrix element $\int i\vec{\alpha}$ would be of order $v_{\text{nucleon}}/c \cong 0.1$. It is interesting to note that all matrix elements involved in the Sb^{124} β_1 transition are considerably reduced. Compared to unique β transitions ($\Delta I = \pm 2$), where $|\int B_{ij}|/R$ is of the order ~0.1, the $\int B_{ij}$ involved in the Sb^{124} β_1 transition is reduced by a factor of about 10. The reduction of $\int B_{ij}/R$, however, is orders of magnitude smaller than the reduction of the other matrix elements. The cause of the unusual predominance of the $\int B_{ij}$ matrix element seems to be a selection rule effect as suggested by Kotani[2] and by Morita and Morita.[3]

The author is greatly indebted to Professor H. Schopper for communicating the β-γ circular polarization correlation results of reference 6 prior to publication and for illuminating discussions. He also wishes to express his gratitude to K. Alder for many suggestions and stimulating discussions.

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†Work supported by the U. S. Atomic Energy Commission.

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[1]E. J. Konopinski and E. G. Uhlenbeck, Phys. Rev. 60, 308 (1941).

[2]T. Kotani, Phys. Rev. 114, 795 (1959).

[3]M. Morita and R. S. Morita, Phys. Rev. 109, 2048 (1959).

[4]R. M. Steffen, Phys. Rev. Letters 3, 277 (1959).

[5]T. Lindquist and I. Marklund, Nuclear Phys. 4, 189 (1957).

[6]G. Hartwig and H. Schopper, following Letter [Phys. Rev. Letters 4, 293 (1960)].

[7]For this computation the following values of the coupling constants (obtained on the basis of the ft values of 0-0 transitions) were used: $C_V = g = (1.41 \pm 0.01) \times 10^{-49}$ erg cm^3, or $C_V = g = 2.97 \times 10^{-12}$ in units $\hbar = m = c = 1$, and $C_A = -1.25\, C_V$.

β -γ CIRCULAR POLARIZATION CORRELATION OF Sb^{124}

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The first forbidden $3^- - 2^+$ transition of Sb^{124} with an end-point energy of 2.3 Mev is of special interest as it exhibits a considerable deviation from an allowed spectrum and a large β -γ anisotropy which cannot be explained by an accidental cancellation of matrix elements as in the decay of RaE. The classical experiments (ft value, shape, β -γ anisotropy) alone do not allow one to determine the 4 matrix elements $\int\vec{r}$, $\int i\vec{\sigma}\times\vec{r}$, $\int i\vec{\alpha}$, and B_{ij} which can contribute to a $\Delta I=\pm 1$, yes transition. It will be shown that a unique determination of these quantities can be achieved by measuring the β -γ circular polarization correlation. The set of matrix elements derived from the experimental results is in agreement with the "modified B_{ij} approximation" requiring B_{ij} matrix elements much larger than all the others. The transition of Sb^{124} investigated here is the first uniquely established example of the "selection rule effect" which has been discussed by several authors.[1,2]

The experimental setup was very similar to that used in previous measurements of the circular polarization of γ rays following allowed β decay.[3] Compton forward scattering from a magnetized iron cylinder was used to detect the γ polarization which is given by $P_\gamma=\omega(v/c)\cos\theta$, where v and c are the velocity of the electrons and of light, respectively, θ is the angle between electron and γ momentum, and ω is the symmetry coefficient.[4] For allowed transitions ω is only a function of the matrix elements and the spins whereas in forbidden transitions it depends also on the total electron energy W (measured in units of m_0c^2) and θ. As ω changes considerably if θ is varied, the instrumental angular spread must be kept as low as possible. To achieve this, three quarters of the scattering magnet was filled with lead, although the coincidence counting rate is decreased appreciably by this procedure. As in addition only a small region of the β spectrum can be used (a second β spectrum starts at $W_0=4.1$), about two months of continuous data collecting were required for one measurement in order to get sufficient statistical accuracy.

The experimental results are presented in Table I. Measurements were performed at two different angles and for two energy ranges. The upper range extending from $W=3.8$ to $W=5.5$ ($\bar{W}=4.3$) contains only a very small contribution from the second β spectrum, whereas in the lower energy range ($W=3.0$ to $W=3.8$) the coincidence rate due to the low-energy branch amounts to about 30%. As the details of this second transition are not known, it is difficult to infer quantitative results from the measurements at the lower β energy. They show, however, qualitatively that ω does not depend strongly on the β energy. On the other hand, ω changes rapidly with θ. Therefore the most accurate determination of the matrix elements will be achieved if measurements at different angles but at a fixed energy are used rather than by taking advantage of the energy dependence.

Table I. Experimental results for $\langle\omega\cos\theta\rangle_{\mathrm{av}}$.

$\bar{W}$	4.3	3.2
$\bar{\theta}=126°$	0.226 ± 0.046	0.190 ± 0.060
$\bar{\theta}=160°$	-0.107 ± 0.035	-0.094 ± 0.043

The equations relating the observed quantities ω and ϵ (the β -γ directional correlation coefficent) to the matrix elements are rather complicated[1,2] but a solution can be found easily in the following way. The difference $W_{\beta\gamma}(160°)\omega(160°) - W_{\beta\gamma}(126°)\omega(126°)$ depends only on one combination of matrix elements i.e., $2x-u$. From the experiments at $\bar{W}=4.3$, one deduces $2x-u=0\pm0.19$. Assuming that $2x=u$ is rigorously valid, one can eliminate x from all equations, leaving the two parameters $\zeta_1=Y+(u-x)W_0/3$ and u. In a ζ_1 -u plane each experimental value determines a conic section (Fig. 1). The intersections of the curves for $\omega(\theta=160°)$ and $\omega(\theta=126°)$ give two sets of solutions but only one is consistent with the β -γ anisotropy.[5]

Within the experimental errors the three curves intersect at one point proving that $2x=u$ is a good approximation.[6] A better intersection than shown in Fig. 1 can be achieved by putting $2x-u=+0.1$ although this change is still within the experimental uncertainties. It may be pointed out that all these calculations were performed by aver-

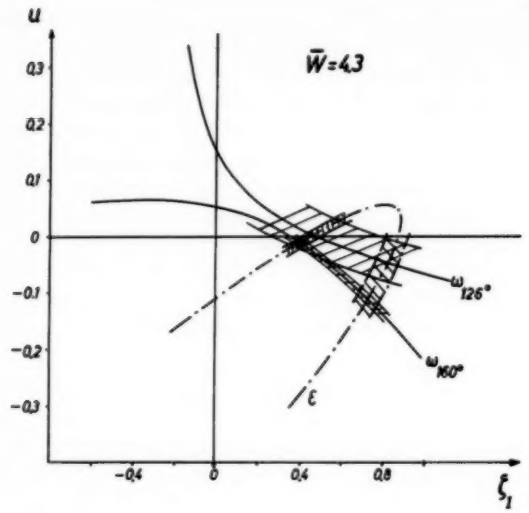

FIG. 1. Graphic determination of the parameters u and ζ_1 from the measured quantities $\omega_{160°}$, $\omega_{126°}$, and ϵ.

aging over the finite sizes of source, β counter, and scattering magnet and over the energy and by taking into account Coulomb corrections. A complete description of these calculations and of the experimental details will be given elsewhere.

In conclusion the following ratios of matrix elements (compared to the standard matrix element B_{ij}) can be inferred:

$$Y = 0.42 \pm 0.1,$$

$$u = -0.01 \pm 0.02,$$

$$x = 0.05 \pm 0.05.$$

The absolute value of B_{ij} can be derived from the ft value by taking into account the results for Y, u, and x and is found to be $|B_{ij}|^2 = (8.0 \pm 0.8) \times 10^{-10}$ sec^{-1}. Thus all matrix elements are known. It turns out that the B_{ij} matrix element is much larger than the momentum-type matrix elements $\int\vec{r}$ and $\int i\vec{\sigma}\times\vec{r}$ and is of the same order of magnitude as the relativistic matrix element $\int i\vec{\alpha}$. Moreover it is found that $\int i\vec{\alpha}$ is in phase with B_{ij}. The unusual enhancement of B_{ij} can be explained by the "selection rule effect" as discussed by Morita and Morita[1] and by Kotani.[2]

The results obtained here can be illustrated by comparing them with some special cases investigated by Kotani.[2] In Fig. 2, $\omega\cos\theta$ is plotted as a function of $\cos\theta$. Curve I corresponds to a

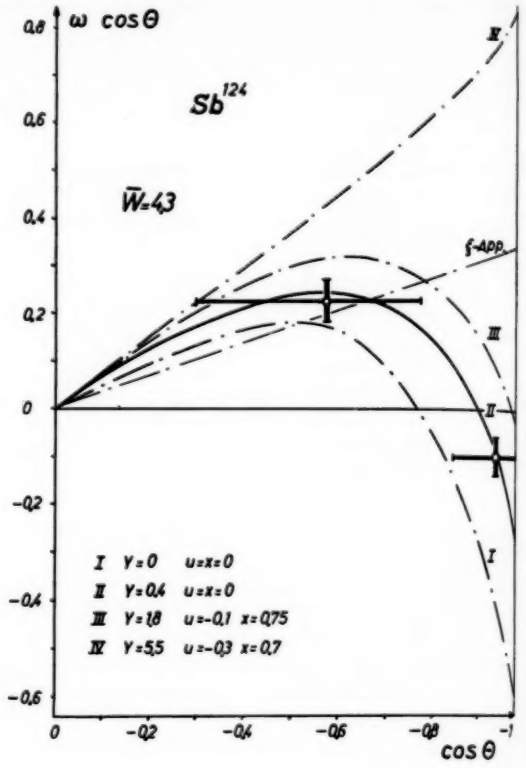

FIG. 2. Comparison of experimental and theoretical values of $\omega\cos\theta$. Curve I: "unique" transition (only B_{ij} contributes). Curve II: modified B_{ij} approximation. Curve III: intermediate case. Curve IV: cancellation of matrix elements.

"unique" decay (B_{ij} contributes only) and curve IV to a cancellation of matrix elements as in the decay of RaE. Another curve shows the "ξ approximation" which is valid in most first forbidden decays. The experimental results clearly exclude all these possibilities and are reproduced only by the "modified B_{ij} approximation" which requires large B_{ij} but $Y \neq 0$.

We would like to thank Dr. H. Appel and Mr. Blatter for their help in setting up the equipment and taking the data and Dr. Morita and Dr. Kotani for communicating their results to us prior to publication and for valuable discussions. We are indebted to the Bundesministerium für Atomkernenergie und Wasserwirtschaft for supporting this work.

[1]M. Morita and R. S. Morita, Phys. Rev. 109, 2048

(1958).
[2]T. Kotani, Phys. Rev. 114, 795 (1959).
[3]H. Schopper, Phil. Mag. 2, 710 (1957); and Nuclear Instr. 3, 158 (1958).
[4]The notation of Kotani[2] is used throughout this paper.
[5]We are greatly obliged to Professor R. M. Steffen for enabling us to use his results prior to publication and for helpful discussions. According to his measurements $\epsilon(p^2/W)^{-1} = -0.09 \pm 0.001$ at $W = 4.3$.
[6]A similar curve could be derived from the shape factor $C(W)$. However, we did not add this curve as C is not known with reliability.

CIRCULAR POLARIZATION OF INTERNAL BREMSSTRAHLUNG ACCOMPANYING β DECAY

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As a consequence of parity nonconservation in weak interactions the particles emitted in β decay possess definite helicities. This is true even for second order processes. In a previous paper[1] it was shown that the photons emitted together with electrons and neutrinos are circularly polarized. For the K capture of A^{37} complete polarization (with positive helicity) was found[2] for the internal bremsstrahlung. The circular polarization of photons with negative helicity accompanying the β decay of Y^{90}, however, showed some deviations from the theoretical expectation.[1] This might be explained by systematic experimental errors or by the circumstance that the decay of Y^{90} is unique forbidden whereas the theory has been developed for allowed transitions only. In order to clarify the situation, the circular polarization of internal bremsstrahlung for different types of β transitions (allowed, first forbidden, unique forbidden) was investigated and all corrections were determined as accurately as possible.

The experimental setup was very similar to that used previously.[1] Compton forward scattering from magnetized iron was used to measure the circular polarization. The inner diameter of the scattering magnet (18 cm) and, at the same time, the length of the central lead absorber (14 cm) were increased in order to achieve a more effective absorption of the direct radiation. The electrons were stopped in paraffin so that the contribution from external bremsstrahlung was kept low. No correction was applied as this influence is negligible compared to statistical errors.

The difficulties in measuring the circular polarization of internal bremsstrahlung are twofold. The yield is very small ($\sim 10^{-3}$ photon/β decay) and thin sources must be used to avoid external bremsstrahlung produced in the source. This requires long counting times in order to get sufficient statistical accuracy. Secondly the spectrum of the bremsstrahlung drops very rapidly with increasing quantum energy. This circumstance, peculiar to bremsstrahlung, makes some corrections much more important than in other experiments. For example, the scattered photons degraded in energy have to compete with the direct radiation which penetrates the central lead absorber and has higher energy and hence a larger primary intensity.

In calculating the γ polarization from the measured change in counting rate (produced by reversing the magnetization), corrections were applied (a) for the elastic scattering of γ rays from the iron cylinder, (b) for Compton scattering from the copper of the magnetization coil, and (c) for photons penetrating the central lead absorber. These corrections are small for low photon energies but amount to about 30% at 1.5 Mev. Apart from these corrections the calculations were performed in the usual way.[3]

In order to avoid the production of external bremsstrahlung the sources and their backings had to be thin. Hostaphan foils (2 mg/cm^2) were used as backings. The sources were thinner than 2 mg/cm^2 and possessed activities of a few mC. A special procedure had to be used to prepare RaE sources.[4] RaE was deposited on Ni foils from RaD solution. As together with RaE some RaF which has a weak γ transition is deposited, a further separation was necessary. This was achieved by evaporating the RaE in a hydrogen stream and condensing it on cooled Al foils (~ 10 mg/cm^2) used as target backings in this case. Due to the short half-life of RaE, several sources had to be produced.

The experimental results for the three decays

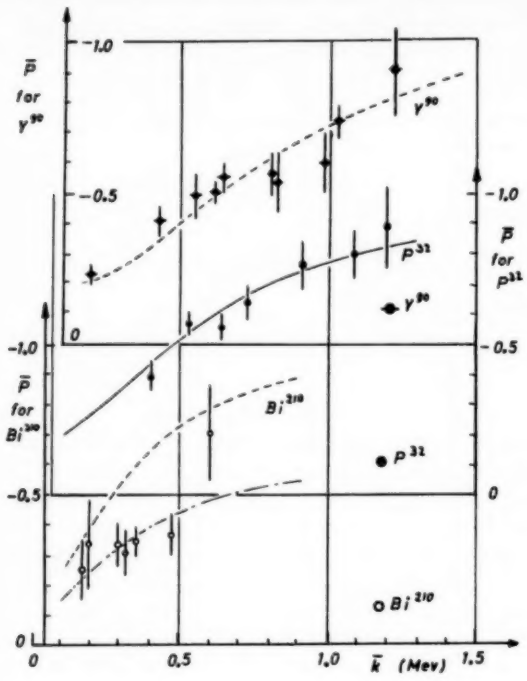

FIG. 1. The circular polarization P of internal bremsstrahlung as a function of the average photon energy $\bar{k}$ for the decays of P^{32} (allowed), RaE (first forbidden), and Y^{90} (unique forbidden). The full and dashed curves are calculated for allowed decays with $d=-1$, whereas for the long dashed curve of RaE the value $d=-0.62$ was used.

investigated are shown in Fig. 1. Different photon energies were observed by selecting the pulse heights with a single-channel discriminator. In order to compare the measured polarizations with theory, the polarization was calculated from the formula

$$P = dJ_2(k)/[J_1(k)+J_2(k)],$$

which has been derived by several authors.[5-7] J_1 and J_2 are functions of the photon energy k and are obtained by an integration over the electron spectrum. The quantity d is a combination of coupling constants and has for allowed transitions the value -1 for a V-A interaction with maximum violation of parity conservation. The polarizations plotted in the figure were evaluated by averaging over the finite energy resolution of the scintillation counter and by taking into account the presence of the Compton peak in the pulse-height distribution of a monoenergetic γ ray. Of course, these corrections must be applied to the measured polarizations and not only to the intensities. Details of these calculations will be published elsewhere.

The polarization of the allowed decay of P^{32} is in good agreement with the theoretical prediction, confirming nonconservation of parity in a second order process. As the theory for forbidden decays has not yet been developed, we compare the experimental results for Y^{90} and RaE with polarizations calculated for allowed decays. It turns out that the polarization of the unique forbidden decay of Y^{90} is in agreement whereas the results for RaE are consistently lower. Similar results were obtained for the longitudinal electron polarization. This similarity, however, is not trivial as the electron and γ polarizations need not be proportional to each other. Since a detailed theory is lacking, we tried to fit the results for RaE by an allowed curve, adjusting the parameter d, and obtained $d=-0.62\pm0.08$. This value is somewhat lower than that inferred from the electron polarization.[8]

We are indebted to Professor Strassmann for enabling us to produce the RaE source in his institute. It is a pleasure to thank Professor Bauer for putting at our disposal the electronic computer Zuse. We are obliged to the Bundesministerium für Atomkernenergie for supporting this work financially.

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(α, t) REACTIONS NEAR $Z = 28$†

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Observation of the charged-particle spectrum resulting from the bombardment of vanadium by 43-Mev alpha particles shows that there is an appreciable cross section for the (α, t) reaction at small angles. It therefore seemed of interest to investigate some (α, t) spectra and angular distributions in order to study the reaction mechanism. The equipment and experimental procedure have been described previously.[1] The spectra of tritons from ${}_{25}Mn^{55}(\alpha, t)_{26}Fe^{56}$, ${}_{27}Co^{59}(\alpha, t)_{28}Ni^{60}$, ${}_{29}Cu^{65}(\alpha, t)_{30}Zn^{66}$, $Rh^{103}(\alpha, t)Pd^{104}$, and $Ta^{181}(\alpha, t)W^{182}$ and of deuterons[2] inelastically scattered from Rh and Ta are shown in Fig. 1.

It is strikingly apparent that the $Mn^{55}(\alpha, t)Fe^{56}$ ground-state transition does not proceed with measurable intensity. This is precisely what would be expected if the (α, t) reaction were a stripping process, wherein a proton is captured by the target nucleus, the angular distribution of the tritons acting as an indicator of the angular momentum of the captured proton. The $Mn^{55}(\alpha, t)Fe^{56}$ ground-state transition would then be forced, by conservation of angular momentum, to proceed by capture of an $f_{5/2}$ proton and would, accordingly, be j-forbidden. Similar considerations would lead one to expect a strong $Co^{59}(\alpha, t)Ni^{60}$ ground-state transition in agreement with the experiment. The transition to the

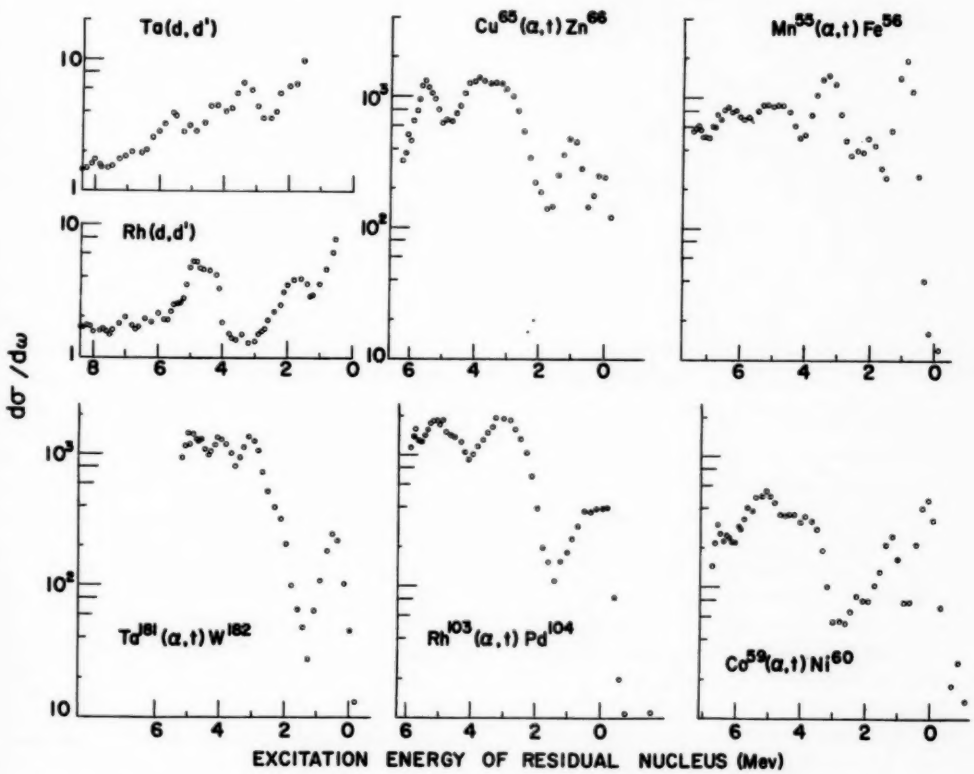

FIG. 1. Triton spectra from the (α, t) reactions on Mn^{55}, Co^{59}, Cu^{63}, Rh^{103}, and Ta^{181}, and deuteron spectra from inelastic scattering on Rh^{103} and Ta^{181} (taken from reference 2). The statistical error on the experimental points varies from 3% to 5% in the case of (α, t) spectra. The triton spectra were obtained at 15° lab, the deuteron spectra at 30° lab. The energies are the excitation energies of the residual nucleus.

2^+ 1-Mev level in Zn^{66} is, however, much stronger than the $Cu^{65}(\alpha,t)Zn^{66}$ ground-state transition.

A sharper test of the "direct" character of the (α,t) reaction mechanism can be obtained by a study of angular distributions of the tritons. In making a detailed study of this matter we cannot proceed from the outset by using the simple Butler[3] theory, since one cannot a priori assume it applicable, at least without serious modifications. One can however proceed empirically by measuring the angular distributions in some cases in which the l value is known with some confidence from shell-model considerations. If this procedure leads to a consistent picture for a number of (α,t) angular distributions the original assumption of a "stripping" type (α,t) process will have been confirmed.

The $V^{51}(\alpha,t)Cr^{52}$, $Fe^{57}(\alpha,t)Co^{58}$, and $Co^{59}(\alpha,t)Ni^{60}$ ground-state reactions (if they proceed by stripping) must involve $l=3$ transitions in contrast to the $l=1$ transition expected in $Cu^{65}(\alpha,t)Zn^{66}$. The angular distributions for Co^{59} and Cu^{65} are shown in Fig. 2, where differential cross sections are given in relative units multiplied by $(k^2+\alpha^2)^2$, where $\vec{k}$ is the momentum-transfer vector and α is a constant determined from the binding energies. Experimentally this is apparently a reasonably satisfactory form factor. The curves $j_1(kR)^2$ and $j_3(kR)^2$ (for $R=7$ fermis) are also shown and are seen to resemble, if not fit, the angular distributions from Cu^{65} and Co^{59}, respectively. The angular distributions of V^{51} and Fe^{57} are quite similar to that of Co^{59}. Since the angular shape of the transition to the first excited (2^+) state of Ni^{60} closely resembles the one to the ground state of Ni^{60} we conclude that it has predominantly $l=3$. Further, since the addition of an $f_{7/2}$ proton can only reach the ground state of Ni^{60}, it must involve the transfer of an $f_{5/2}$ nucleon. The angular distribution of the $Mn^{55}(\alpha,t)Fe^{56}$ reaction to the 840-kev 2^+ level clearly has an $l=3$ shape. The transition to the group observed near 3.5 Mev in Fe^{56} is shown to have a considerable $l=1$ admixture.

It is clear that more (α,t) angular distributions must be measured in order to refine the kind of analysis outlined above. The accumulated evidence so far seems to point strongly to a process of the "stripping" type as the reaction mechanism. Therefore it seemed of interest to analyze a few of the gross-structure groups observed in the Mn^{55}, Fe^{57}, Co^{59}, and Cu^{65} reactions to see whether an analysis of the type used by Schiffer et al.[4] for the gross structure observed in (d,p) reactions would give results consistent with shell-model expectations. The result for the $Mn^{55}(\alpha,t)Fe^{56}$ reaction to the 3.5-Mev group is in excellent agreement with expectations. A preliminary analysis for the other gross structures was also consistent with qualitative shell-model predictions. It seems therefore plausible that the gross structure in (α,t) reactions may be interpreted in terms of single-particle states of the proton.

The gross structure of the triton spectra from Pd^{104} and W^{182} are seen to be very similar[5] to that of the inelastic deuteron spectra from Rh^{103} and Ta^{181}. This suggests that an interpretation in terms of single-particle excitation for the gross structure observed in the spectra of inelastically scattered protons,[6] deuterons, and

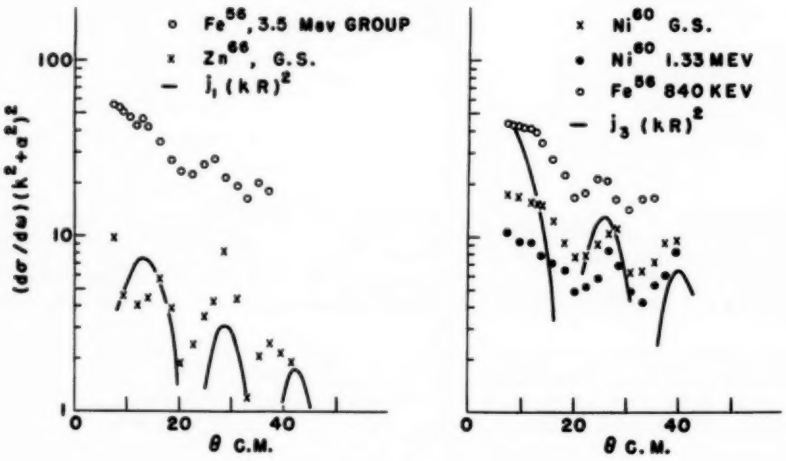

FIG. 2. Angular distributions of $Co^{59}(\alpha,t)Ni^{60}$ ground state, $Co^{59}(\alpha,t)Ni^{60}$ 1.33-Mev level, $Mn^{55}(\alpha,t)Fe^{56}$ 820-kev level, $Mn^{55}(\alpha,t)Fe^{56}$ 3.5-Mev group, and $Cu^{65}(\alpha,t)Zn^{66}$ ground state. The angles are in the center-of-mass system. The relative cross sections have been multiplied by $(k^2+\alpha^2)^2$, where $\vec{k}$ is the momentum-transfer vector and α a constant. The curves $j_1(kR)^2$ and $j_3(kR)^2$ are shown for $R=7$ fermis.

alpha particles[7,8] may be possible.

I am indebted to Dr. M. H. Macfarlane, Dr. B. J. Raz, and Dr. B. Zeidman for discussions and suggestions in connection with this work, to W. J. O'Neill for technical assistance, and to W. Ramler and the cyclotron group for their cooperation.

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POSSIBILITY OF A TEST OF THE CONSERVED VECTOR CURRENT THEORY IN THE $A=8$ POLYAD*

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The measurement of the β-α angular correlation function in the beta decay of Li^8 and B^8 has been proposed[1] as a test of the conserved vector current theory (C.V.C.T.) of beta decay.[2] In order to interpret recent experimental information,[3] a knowledge of the unknown $M1$ and $E2$ transition probabilities for the transition from the first $J=2$, $T=1$ state in Be^8 to the first excited state ($J=2$, $T=0$) is necessary.[4] It is especially interesting to know how much of the $M1$ transition probability in question is due to the spin part of the magnetic moment operator since only that part gives an enhancement of the corresponding second forbidden matrix element in beta decay, compared to the prediction of the Fermi theory (F.T.).

This Letter reports on an attempt to determine the quantities of interest by an intermediate-coupling calculation.[5] The over-all agreement of Kurath's[6] results with the experimental level scheme in the $A=8$ polyad, his calculation of the $M1$ widths of the 17.6-Mev state[7] in Be^8, and especially the close agreement of his calculation[8] of the magnetic moment of Li^8 with observation seem to encourage such an attempt.

Using the standard notation as defined in reference 6, we carried through the calculation for force mixtures of the Serber, Kurath, and Rosenfeld types, for values of L/K between 5.3 and 8.3 and of a/K between 0 and 5. We calculated the energy spectrum for $J=2$ states, the magnetic moment of Li^8, the logft of the beta decay, the $M1$ and the $E2$ transition amplitudes, and corrections to the magnetic moment and the $M1$ amplitude. These corrections are due to the presence of spin-orbit and exchange forces and have to be introduced in order that the Hamiltonian be gauge-invariant.[9,10] It is hoped that the calculation of these corrections provides an order of magnitude estimate of unknown terms in the magnetic moment operator, such as exchange moments, etc.

The relative position of the low-lying $J=2$ states obtained with the three different types of forces and different values of L/K differ very little from each other over the whole range of a/K if one adopts the scale parameter K properly. The attempt to fit simultaneously the magnetic moment and logft fails the Serber forces. This is due to the fact that these forces produce a very strong admixture of $[31]D$ states to the $J=2$, $T=1$ state (which in L-S coupling is $[31]^3P$). These states contribute considerably to the magnetic moment and very little to the beta transition. This result is in agreement with the observation that one cannot reproduce the level scheme of Li^8 using Serber forces. We will therefore not consider Serber forces in the following.

The spread of the other curves obtained for μ,

logft, and the $M1$ amplitude when plotted versus a/K is surprisingly small and confirms the reliability of the estimate. Since the operator for the beta decay seems to be much better known than the magnetic moment operator, we tried to fit[11] logft = 5.7 by choosing a/K properly.[12] In this way one obtains for each set of parameters a prediction of the magnetic moment. The deviations of these numbers from the observed value exceed in no case 20%. The range of values of a/K for such a fit is 1.25, ... 2.50, in agreement with Kurath's calculations. From these values of a/K and the calculated curves for $\Lambda(M1)$, the transition strength in the definition of Lane[12] as shown in Fig. 1, one obtains $0.40 \leq \Lambda(M1) \leq 0.50$, or for the $M1$ width 3 ev $\leq \Gamma(M1) \leq 4$ ev.

Figure 2 shows the composition of the $M1$ matrix element for a typical case, $L/K = 6.8$ and Kurath force mixture. In this case, the a/K value which fits logft is 2.00. The matrix element is defined so that $\Lambda(M1) = \frac{3}{8}\langle \| M1 \| \rangle^2 = 0.5$ in this case. Only the isovector part of the magnetic moment operator,

$$\vec{\mu}_v(i) = (e\hbar/2Mc)[-\tfrac{1}{2}\tau_z^{(i)}\vec{l}(i) + \tfrac{1}{2}\tau_z^{(i)}(\mu_n - \mu_p)\vec{\sigma}(i)],$$

contributes to the transition, and one sees that orbital and spin part give contributions with the same sign and similar in magnitude. This statement holds approximately for all cases calculated. The dashed line indicates the value one obtains for the $M1$ matrix element if one neglects the anomalous moments, i.e., replaces $\vec{\mu}_v$ by $(e\hbar/2Mc)[-\frac{1}{2}\tau_z^{(i)}\vec{l}(i) - \frac{1}{2}\tau_z^{(i)}\vec{\sigma}(i)]$.

The smallness of the spin-contribution can be easily understood by the observation that the final state is in pure L-S coupling a $[4]\,^1D$ state. For $a/K = 2.5$ the admixtures of other states are very small (largest admixture 15% in amplitude). Since the $J = 2$, $T = 1$ states do not contain the symmetry class [4], the operator $(\sigma\tau)$ has vanishing matrix elements between any of the $T = 1$ states and the $[4]\,^1D$ state. This statement does not hold, however, for the orbital part of the operator. For later use we note that

$$3.2 \leq \langle \| l \| \rangle / \langle \| \sigma \| \rangle \leq 4.13,$$

and also that Λ varies practically as the square of a/K and is therefore extremely sensitive to this choice. If one chooses slightly higher values for a/K, one obtains bigger Λ's and a larger predicted asymmetry in the β-α angular correlation. At the same time, however, one would predict a larger Gamow-Teller matrix element than observed.

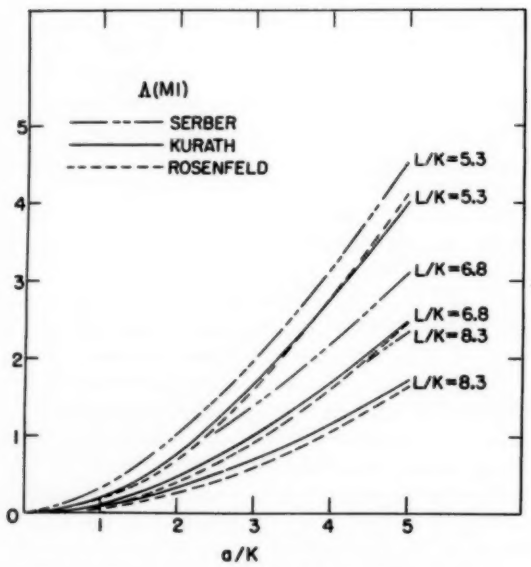

FIG. 1. The strength of the $M1$ transition as a function of a/K for various choices of parameters as indicated in the figure. E in Mev, $\Gamma(M1)$ in ev = $2.76 \times 10^{-3} \times E^3 \times \Lambda(M1)$.

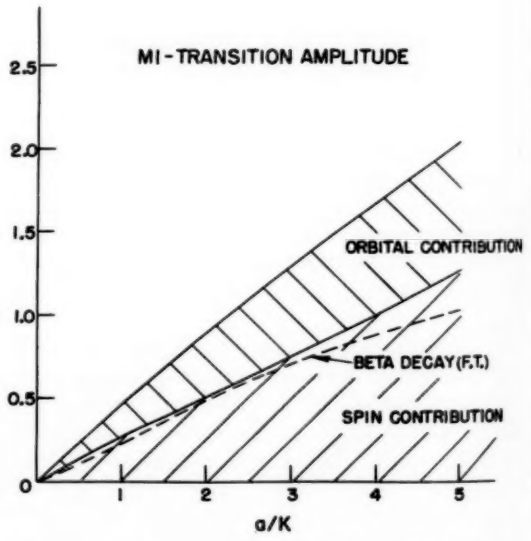

FIG. 2. The composition of the $M1$ matrix element as a function of a/K for a typical case: Kurath force mixture and $L/K = 6.8$. The dashed line gives the matrix element which one has to use in the Fermi theory of beta decay; the uppermost line (the total $M1$ amplitude) gives the matrix element in the conserved vector current theory.

As an order of magnitude estimate of additional contributions to the $M1$ transition amplitude and the magnetic moment of Li^8, the contributions due to the presence of a spin-orbit-coupling term in the Hamiltonian were calculated. The magnetic dipole operator corresponding to this term is given in reference 9:

$$\vec{\mu}_{\text{add}}(i) = \frac{e\hbar\gamma}{2Mc}\tfrac{1}{2}(1-\tau_z^{(i)})\left\{\left(\vec{\sigma}_i \cdot \frac{\vec{r}_i}{r_i}\right)\frac{\vec{r}_i}{r_i} - \vec{\sigma}_i\right\}.$$

Here, $\gamma = aM\langle r^2\rangle/\hbar^2 \approx -\frac{1}{2}$ for $a = -2.5$ Mev. This gives a correction of +10% at $a = -2.5$ Mev, +25% at $a = -5$ Mev to the magnetic moment and of -15% at $a = -2.5$ Mev, -25% at $a = -5$ Mev to the $M1$ amplitude. This is within the uncertainty of our fit. It changes, however, the above statement to $1.6 \leq \langle \|l\| \rangle / \langle \|\sigma\| \rangle \leq 5.0$. No attempt was made to estimate the contribution due to exchange forces[10] since there are radial integrals involved on which the expression seems to depend sensitively and about which no information is available.

For the parameters of interest, calculation of the ratio of the transition strengths for the $E2$ and $M1$ transitions gave values smaller than 6×10^{-4}. Here, as in the preceding estimate of γ, we assumed $\langle r^2\rangle = \frac{1}{2}\times10^{-25}$ cm^2. The actual value of the ratio of the reduced matrix elements depends, however, very sensitively on the parameters used; in particular, it can be positive or negative.

With this information we are now in a position to discuss the prediction of the β-α angular correlation functions in B^8 for F.T. and C.V.C.T., assuming zero anisotropy in the Li^8 correlation function. One obtains the following results.

(a) Fermi theory:

$$W(\theta, \text{F.T.}) = 1 - 0.008\frac{\langle \|l+\sigma\| \rangle}{\langle \|\sigma\| \rangle} P_2(\cos\theta)\frac{P^2}{WW_0}\left\{1 + 2\frac{\langle \|E2\| \rangle}{\langle \|l+\sigma\| \rangle (\frac{2}{5})^{1/2}}\right.$$

$$\left. - 1.44\frac{\langle \|ip_1[3\sigma_i x_k + 3x_i\sigma_k - 2(\vec{\sigma}\cdot\vec{r})\delta_{ik}]\| \rangle}{(\frac{3}{5})^{1/2}\langle \|l+\sigma\| \rangle}\right\},$$

where the reduced matrix elements $\langle \|O\| \rangle$ are dimensionless quantities.

(b) Conserved vector current theory:

$$W(\theta, \text{C.V.C.T.}) = 1 - 0.008\frac{\langle \|l+4.7\sigma\| \rangle}{\langle \|\sigma\| \rangle} P_2(\cos\theta)\frac{P^2}{WW_0}\left\{1 + 0.56\frac{\langle \|E2\| \rangle}{(\frac{3}{5})^{1/2}\langle \|l+4.7\sigma\| \rangle}\right\}.$$

Here, W_0 is the maximum, W the actual energy of the electrons in units $\hbar = c = m = 1$, P their momentum. In both cases the quantities in brackets have a small energy dependence, which for practical purposes, however, can be neglected.

We know from our calculation that $\langle \|l+\sigma\| \rangle / \langle \|\sigma\| \rangle \approx 1.6, \ldots 5$; $\langle \|l+6.7\sigma\| \rangle / \langle \|\sigma\| \rangle \approx 6, \ldots 9.5$; and $\langle \|E2\| \rangle / |\langle \|l+\sigma\| \rangle (\frac{3}{5})^{1/2}| \lesssim 0.03$.

If we neglect the $E2$ terms in both expressions for the moment, we obtain

$$W(\theta, \text{F.T.}) = 1 + \alpha P_2(\cos\theta),$$

where $-0.04 \leq \alpha \leq -0.01$,

$$W(\theta, \text{C.V.C.T.}) = 1 + \alpha P_2(\cos\theta),$$

where $-0.08 \leq \alpha \leq -0.05$.

This is to be compared with an experimental finding of $\alpha \approx (-1.5 \pm 3)\%$, where the quoted error is purely statistical, and includes no estimates of possible systematic errors.[3] This result favors the F. T. over the C.V.C.T., though not strongly.

In conclusion it must be stressed, however, that the $A = 8$ polyad does not seem well suited for a critical test of the C.V.C.T. because of the large contribution of the orbital angular momentum operator to the $M1$ transition amplitude, and because of the well-known fact that our estimate of the $E2$ amplitude may be too small by a factor of three or even ten.

The author would like to thank Mr. E. Nordberg and Mr. F. Morinigo for their help in the numerical calculation. He is greatly indebted to Professor C. A. Barnes, Professor F. Boehm, and Professor R. F. Christy for discussions and to Professor B. Stech of Heidelberg University

for communications. The assistance and interest of Barbara Zimmermann in preparing the program for diagonalizing the matrices is gratefully acknowledged. It is a pleasure to thank the Kellogg Radiation Laboratory for its hospitality.

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†On leave of absence from Institut für Theoretische Physik der Universität, Heidelberg, Germany.

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R PARAMETER IN *p*-*p* SCATTERING AT 142 MEV

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As part of the Harwell program on nucleon-nucleon scattering a measurement has been made of the Wolfenstein R parameter in proton-proton triple scattering at 142 Mev.

The proton beam whose direction of polarization was vertical, and for which $P_1 = 0.46 \pm 0.01$, was passed along the axis of a solenoid of such integrated field strength that the proton spins were rotated into the horizontal plane. The direction of horizontal polarization could be reversed by reversing the direction of the magnetic field in the solenoid. The beam was then focussed by a quadrupole pair on to a 6-inch long liquid hydrogen target.

The horizontal transverse polarization of protons scattered at angle θ in the horizontal plane (i.e., in the plane containing the direction of polarization and the incident vector) was then measured by scattering in the up-down plane from targets of beryllium of analyzing power P_3. Protons scattered up and down were detected in two two-counter telescopes placed in coincidence with a counter immediately in front of the beryllium analyzer. Altogether ten counter telescopes were used, which allowed measurements to be made at five different scattering angles θ simultaneously, in a self-monitoring system. The asymmetry RP_1P_3 in the scattering from the beryllium analyzers was determined by comparison of the counting rates in a given counter telescope for the two directions of solenoid field (or proton spin).

The over-all analyzing power P_1P_3 was determined by moving each telescope to $\theta = 0$ and reducing the energy of the beam with an absorber so that it corresponded to the energy of protons scattered from the hydrogen target at the appropriate angle θ.

The R experiment, particularly when it is self monitoring and performed with a solenoid, is very little subject to systematic errors. There is, however, one serious possibility for error which arises from the focussing properties of the solenoid for protons. It is necessary to position the solenoid extremely carefully so that the mean position of the beam at the hydrogen target does not move appreciably when the solenoid current is reversed. In the present experiment, the movement was not more than 0.2 mm, which called for corrections much less than the final standard errors on all the measurements.

As a final check on the systematic errors in the measurements, the experiment was repeated with an unpolarized incident beam, and the asymmetries were found to be consistent with zero in this case.

Table I. Measured asymmetries and derived values of R.

θ c.m.	Asymmetry RP_1P_3	P_1P_3	R
24.0	-0.0421 ± 0.0095	0.188	-0.224 ± 0.051
32.7	-0.0351 ± 0.0088	0.173	-0.203 ± 0.051
45.7	-0.0249 ± 0.0044	0.140	-0.178 ± 0.031
54.4	-0.0252 ± 0.0050	0.119	-0.212 ± 0.042
67.2	-0.0187 ± 0.0035	0.088	-0.213 ± 0.040
76.1	-0.0100 ± 0.0043	0.068	-0.147 ± 0.063
84.0	-0.0084 ± 0.0080	0.059	-0.142 ± 0.136
90.0	$+0.0054 \pm 0.0064$	0.049	$+0.110 \pm 0.131$

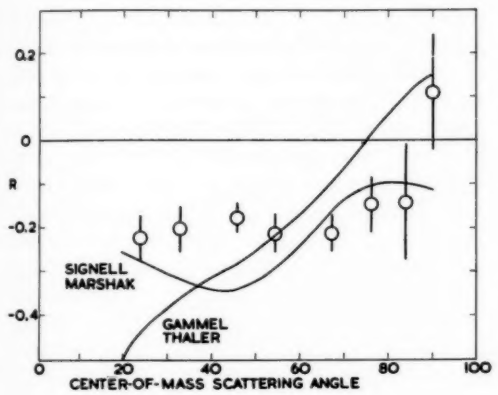

FIG. 1. R parameter in p-p scattering at 142 Mev, with earlier theoretical estimates by Signell and Marshak and by Gammel and Thaler.

The values of asymmetries, analyzing powers, and of R are given in Table I. The results are shown in Fig. 1, together with predictions from the Signell-Marshak[1] and the Gammel-Thaler[2] potentials. These are both in qualitative accord with the trend of the experimental points.

Preliminary results have been presented at the Few-Nucleon Conference, London, July, 1959 (unpublished) and at the Ninth Annual International Conference on High-Energy Physics, Kiev, 1959 (unpublished).

A full account of the experiment will be published elsewhere.

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POSSIBLE NEW RESONANCE IN THE π^+-p SYSTEM

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Within the past year a great deal has been learned about the interactions of high-energy negative π mesons with protons. In particular two very striking peaks have been found to exist in the π^--p cross section at lab pion energies of about 600 and 900 Mev.[1-3] Angular distributions of elastically scattered pions in this energy range have been reported.[2,4] However, experimental data for π^+-p scattering in the same energy range are not nearly so complete. Total cross-section measurements[1,5] have indicated that in this energy region the π^+-p cross section reaches a minimum at about 650 Mev, thereafter rising slowly to the broad peak near 1.3 Bev.[5,6]

In the present note we wish to anticipate future investigations of the π^+-p system by drawing attention to some features of data already available which suggest the existence of a new resonance in the isotopic spin 3/2 (π^+-p) state at an energy of about 850-950 Mev.[7] This evidence further suggests that the favored state has negative parity and total angular momentum 3/2 ($D_{3/2}$). This is precisely the state of angular momentum

and parity assigned to the 600-Mev resonance of isotopic spin 1/2 (the "second" resonance) through the work of Wilson,[8] Peierls,[9] Sakurai,[10] and Stein.[11] This assignment for the second resonance is essential to the arguments presented here and will be assumed throughout. With the notation L_{JT} (L is the orbital momentum, J the total angular momentum, T the total isotopic spin) we suggest that there is a $D_{3/2,3/2}$ resonance as well as the already established[8-11] $D_{3/2,1/2}$ resonance. Theoretical arguments are given for the particular quantum numbers of the suggested resonance, and for the different energies of the two $D_{3/2}$ resonances, at the end of this Letter.

Unfortunately the hypothetical $D_{3/2,3/2}$ resonance cannot be expected to reveal itself in such a dramatic fashion as have the well-established resonances, for the following reasons. First of all at such high energies (near 900 Mev) the maximum permissible cross section for a state having $J=3/2$ is only 30 mb, and this will be further reduced by the occurrence of inelastic processes. Further, since the proposed resonance lies at the foot of the broad peak at 1.3 Bev, the amplitudes for the states composing this latter peak will already be appreciable before the $D_{3/2,3/2}$ resonance has subsided, preventing the appearance of a distinct peak, although one may hope that more refined experiments will reveal some structure in the cross section. (Clearly these observations do not diminish the importance of such a resonance for the understanding of the dynamics of meson-nucleon interactions.) Therefore a measurement is proposed to detect the characteristic PD interference in the recoil proton polarization [see Eq. (3)]. The Saclay group[3] reported π^+-p measurements up to about 850 Mev. Their data, besides displaying sharper and higher π^--p peaks, show both a lower minimum in the π^+-p cross section than previously found[1,5] and an abrupt rise in $\sigma(\pi^+-p)$ in the interval 700-850 Mev. If this curve is continued up to the points of reference 6, a distinct shoulder appears in the cross section.[12] We now discuss the possible structure of this shoulder.

First consider the remarkable behavior of the elastic charge exchange scattering ($\pi^-+p\rightarrow\pi^0+n$) in the 500-800 Mev energy range. Throughout this interval the charge exchange scattering is small and rapidly decreasing, without any sign of a peak, although the elastic scattering (π^--p) displays a sharp peak near 600 Mev.[2] This is rather surprising, since the cross sections are given in terms of the amplitudes $f_{3/2}$ and $f_{1/2}$ (the subscript denotes isotopic spin) by

$$\sigma_{el}^{-}\propto|f_{3/2}|^2+4|f_{1/2}|^2+4\mathrm{Re}(f_{1/2}^{*}f_{3/2}), \quad (1)$$

$$\sigma_{ex}^{-}\propto 2|f_{3/2}|^2+2|f_{1/2}|^2-4\mathrm{Re}(f_{1/2}^{*}f_{3/2}). \quad (2)$$

[The proportionality factor is the same for (1) and (2).] Since $f_{1/2}$ is known to vary in a resonant manner, the smallness and smoothness of the charge-exchange scattering requires that the interference term $\mathrm{Re}(f_{1/2}^{*}f_{3/2})$ be large and positive, a point already noticed by Peierls.[13] Since we are concerned with total cross sections, only states having the same L and the same J survive in the interference term. Now if these $T=3/2$ states were small and nonresonant, one would expect the charge exchange cross section to rise abruptly on the high-energy side of the resonance, which is not the case. Since $D_{3/2,1/2}$ presumably is the most important state in this energy interval, the simplest way to prevent both the occurrence of a peak in σ_{ex} and an abrupt rise above the second resonance is to have the $D_{3/2,3/2}$ phase shift positive and growing rapidly above 600 Mev. (The relative phase of the states $D_{3/2,1/2}$ and $D_{3/2,3/2}$ is expected to be large and near 90° between about 600 and 900 Mev, but it must be rather less than 90° for the present argument to work.) Let us consider the other interference terms. Since the S-wave phase shifts are not too large and probably have opposite signs,[2,14,15] their interference will not help in explaining the behavior of σ_{ex}. Next consider the $P_{1/2}$ interference; α_{11} and α_{31} are both small and negative (we use $\alpha_{2T,2J}$ for the P phase shifts, and reserve $\delta_{2T,2J}$ for the D waves) at 300 Mev,[15,16] and at least α_{31} is negative at 500 Mev.[14] Hence this term will either be small or else have the wrong sign. The $P_{3/2}$ interference term will have the wrong sign as long as α_{13} is positive.[15] In any event α_{33} is varying slowly in the energy region under consideration. Accurate $T=3/2$ D-wave phase shifts have been obtained at 310 Mev by Foote et al.[16] They find that δ_{33} is positive, indicating an attractive $D_{3/2,3/2}$ state, consistent with the present scheme, and a negative $D_{5/2,3/2}$ phase (δ_{35}). The solutions f and g obtained by Willis also have this character.[14] Crittenden et al.[2] report a tentatively negative (and small) δ_{15} at 460 Mev. Thus it seems unlikely that $D_{3/2}$ interference can have the required character. There is some evidence[13] that

the third resonance ($T=\frac{1}{2}$) may be associated with an $F_{5/2,\,1/2}$ state. However, since the proposed resonance is supposed to have an energy comparable to the third resonance, it seems impossible for the $F_{5/2,\,3/2}-F_{5/2,\,1/2}$ interference to be appreciable in the vicinity of the second resonance.

Further, the size of the π^+-p cross section at 900 Mev (≈ 22 mb) is compatible with a dominant $D_{3/2}$ state with an inelastic scattering of about 6 mb. (Since the other states have been ignored, this underestimates σ_{in}.) This is to be compared with $\sigma_{in}=2.8\pm 0.5$ mb at 500 Mev[14] and $\sigma_{in}=10.5\pm 1.5$ mb at 990 Mev.[17] Only a completely absorptive $J=5/2$ wave can have as small a total cross section as 22 mb at 900 Mev.

Now consider the available π^+-p angular distributions. At both 500 Mev[14] and 1.1 Bev[18] the most noticeable feature is the very small amount of scattering in the backwards direction. If the scattering is really strong in a $D_{3/2}$ state around 850-900 Mev, then one should expect some $1+3\cos^2\theta$ component to be detectable in the backwards hemisphere. Now in fact, at 990 Mev, where the "resonance" should still be appreciable, there is in fact an encouraging backwards maximum.[17] Photoproduction angular distributions should show signs of this state, but in the total cross section the proposed resonance probably will not be distinguishable from the third resonance, which occurs at about the same energy.

As mentioned before, the detection of a large negative (with respect to the normal to the plane of scattering) polarization for large scattering angles (forward protons) at an energy of, say, 800 Mev would be good evidence for the picture presented above. This rough choice of energy would be low enough to avoid complications from other states and the appreciable inelastic scattering which accompany the 1.3-Bev peak. It is low enough for the tail of the $P_{3/2,\,3/2}$ resonance to be large and high enough for the proposed $D_{3/2,\,3/2}$ resonance to be important. Keeping only these two states in computing the polarization, one has for the polarization of the recoil proton {the amplitudes f are $[\rho\exp(2i\delta)-1]/2ik$; $0\leq\rho\leq 1$}

$$P_n k^2 d\sigma/d\Omega = 2\sin\theta(9\cos^2\theta-1)\mathrm{Im}(f_{P_{3/2,\,3/2}}{}^{*} f_{D_{3/2,\,3/2}}). \quad (3)$$

(k is the pion momentum in units of $\hbar$.) Note the large $\cos^2\theta$ term. One can see that the other states involved give interference terms which are likely to be small, and if anything, tend to offset the above polarization. [Note that if $D_{3/2,\,3/2}$ is replaced by $D_{5/2,\,3/2}$ then the right-hand side of (3) is generally smaller and more isotropic since $9\cos^2\theta-1$ becomes $\frac{3}{2}(1+\cos^2\theta)$.] The actual magnitude of P_n depends on α_{33}. The proton energy (near 400 Mev) should make the polarization experiment feasible.

The dispersion relation calculations of Sternheimer[19] show that the real part of the forward scattering amplitude for π^+-p scattering vanishes at about 900 Mev pion energy. Such a circumstance is certainly compatible with the existence of a dominant resonance state at this energy (if the real part of the phase shift is 90°, then f is purely imaginary). At this energy the S waves and some of the P waves are probably mostly absorptive, making no contribution to $\mathrm{Re}(f)$, and the cancellation of the amplitudes of the remaining states is quite plausible.

Peierls[13] has interpreted the second and third resonances in terms of two-meson states, utilizing ideas from the isobar model.[20] For instance, for the negative-parity second resonance ($D_{3/2,\,1/2}$) the S-P configuration is supposed to be important. This model had some difficulty in explaining why there was a resonance in the $D_{3/2,\,1/2}$ state but not in the $D_{3/2,\,3/2}$ state. This difficulty is resolved if the point of view of the present note is taken.

Finally it is necessary to consider why the two D-wave resonances should be so far apart (250-300 Mev in the lab; 150-175 Mev c.m.) in energy. As might be expected, this behavior is to be blamed on the 33 resonance. A partial explanation may be contained in the observation that in the partial wave dispersion relations the cross section for the 33 resonance enters with opposite signs for the $T=1/2$ and $3/2$ amplitudes.

The author would like to thank Professor H. A. Bethe for a number of enlightening discussions of this work, and Dr. A. Erwin for supplying some unpublished data.

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FEASIBILITY OF USING HIGH-ENERGY NEUTRINOS TO STUDY THE WEAK INTERACTIONS

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For many years, the question to how to investigate the behavior of the weak interactions at high energies has been one of considerable interest. It is the purpose of this note to show that experiments pointed in this direction, though not quite feasible with presently existing equipment, are within the capabilities of present technology and should be possible within the next decade.

We propose the use of high-energy neutrinos as a probe to investigate the weak interactions.

A natural source of high-energy neutrinos are high-energy pions. Such pions will produce neutrinos whose laboratory energy will range with equal probability from zero to 45% of the pion energy, and whose direction will tend very much toward the pion direction. For example, 1-Bev/c pions will emit neutrinos with an average energy of ~220 Mev in such a way that ~$\frac{1}{2}$ of the neutrinos will fall within a cone of half-angle 7°. For orientation purposes, the mean decay distance for such a pion would be 50 meters.

The best known source of pions is a proton accelerator where the beam is allowed to impinge on a target. Let us assume that we have available a 3-Bev proton beam and 10 000 kilograms of material for sensing a neutrino interaction. We may then estimate the proton flux necessary to produce one interaction per hour with a cross section of σ cm^2. To do this, let us consider the simple setup shown in Fig. 1.

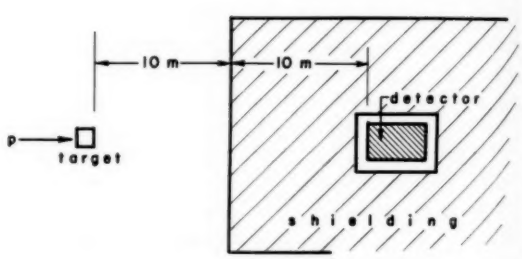

FIG. 1. Proposed experimental arrangement.

Let I be the number of incident protons per unit time, and let, say, $I/10$ charged pions with energy ≥ 2 Bev be produced at the target. These pions emerge in a cone of about 45° half-angle, or in about 2 steradians of solid angle. We now let them travel for a distance of 10 meters before hitting a 10-meter shielding wall in front of the detector. Approximately 10% of the pions will decay with an average neutrino energy of about 400 Mev. Each square centimeter of detector subtends a solid angle of $\frac{1}{4}\times10^{-6}$ steradian. Hence, the high-energy neutrino flux at the detector is $(\frac{1}{10}I)(\frac{1}{4}\times10^{-6})(\frac{1}{10})(\frac{1}{2})\cong 1\times10^{-9}I$. If there are 10 000 kilograms of detector present, the number of events per unit time is given by

$$N\sim(10^7)(6\times10^{23})(10^{-9}I)\sigma=6\times10^{21}I\sigma.$$

For an intensity of $I=5\times10^{12}$ protons/sec $=1.8\times10^{16}$

protons/hr, the high-energy neutrino flux is ~5000 neutrinos sec^{-1} cm^{-2}. With a cross section $\sigma \sim 10^{-38}$ cm^2, the number of counts is $N \sim 1$ per hour in 10 000 kg of detector. The estimate here given is for neutrinos from high-energy pions. There is, as a matter of fact, a much greater flux of lower energy neutrinos from lower energy pions. However, because the neutrino cross section decreases rapidly with decreasing energy, the rate is not likely to be improved by more than a factor of two.

This estimate places the experiment outside the capabilities of existing machines by one or two orders of magnitude. Optimistic estimates for accelerators which are currently under construction, namely the 3-Bev machine at Princeton and the 10-Bev machine at Argonne, indicate that the experiments may be barely feasible in the near future. However, for really quantitative experiments it will be necessary to use high-intensity machines such as the FFAG machine proposed by MURA or the 10-Bev linear proton accelerator discussed by Blewett at Brookhaven. In these machines, one hopes to attain a beam intensity of the order of 10^{15} protons/sec at an energy of about 10 Bev.

The higher energy of the primary beam of protons makes the experiment easier because of the increased multiplicity of pions, the more concentrated forward distribution of the pions, and the increased cross section for neutrino reactions. Balanced against these is the fact that the percentage of higher energy pions that decay in 10 meters is smaller. The net result is likely to give a counting rate per primary proton that probably increases more than linearly with the primary proton energy.

Thus, a high-intensity 10-Bev proton machine with a beam intensity $\sim 10^{15}$ protons/sec may give a counting rate of more than 10^3 per hour, using the experimental setup described above. If that proves to be the case, it is perhaps desirable to have magnetic lenses to analyze and focus the pions so as to obtain more monoenergetic neutrino beams.

I would like to express my gratitude to Dr. T. D. Lee and Dr. C. N. Yang for many stimulating discussions which led to the above proposal.

Note added in proof. The author's attention has been called to a somewhat related paper which has just appeared: B. Pontecorvo, J. Exptl. Theoret. Phys. (U.S.S.R.) 37, 1751 (1959).

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THEORETICAL DISCUSSIONS ON POSSIBLE HIGH-ENERGY NEUTRINO EXPERIMENTS*

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The weak interaction so far most extensively studied is β decay, in which the momentum transfer is of the order of a few Mev. In μ decay and μ capture, momentum transfers of the order of 100 Mev are involved. In the theory of these processes, because of the limited region of momentum transfer studied, the phenomena can be described by a few parameters usually called coupling constants. For larger momentum transfers it is obvious that the weak interactions cannot continue to be described by these constants, because of the clothed structure of the nucleons due to the strong interaction, and also because of the reasonable expectation that the weak interactions, even without the interference of the strong interactions, may not be of the simple four-spinor product form in Fermi's theory.

In the preceding Letter,[1] Schwartz points out that the neutrinos from the decay of high-energy mesons can be used to study weak interactions. We have investigated the theoretical implication of such possible experiments. Efforts are made to separate and dissociate the inferences that can be drawn from different assumptions concerning the weak interactions. In this Letter we report briefly on this work.

1. The identity of the neutrinos. In the processes

$$\pi^+ \to \mu^+ + \nu_1, \qquad (\pi \text{ decay}) \qquad (1)$$

$$\mu^- + p \to n + \nu_2, \qquad (\mu \text{ capture}) \qquad (2)$$

$$Z \to (Z-1) + e^+ + \nu_3, \qquad (\beta^+ \text{ decay}) \qquad (3)$$

it is easy to see that ν_1 and ν_2 are the same par-

ticle. Experimentally it is known that ν_1 and ν_3 both have helicity -1. It is simplest to assume that ν_1 and ν_3 are also the same particle. However, a test of this assumption is clearly desirable. To obtain such a test it is necessary to do some kind of capture experiment on the neutrinos or antineutrinos. For example, if ν_1 and ν_3 are different particles, then the reaction

$$n+\nu_1 \rightarrow p+e^- \tag{4}$$

does not occur.

2. Conservation of leptons. The conservation of leptons can be studied with neutrino capture experiments. For example, if both

$$\nu_1+p \rightarrow \Lambda^0+e^+ \tag{5}$$

and

$$\bar{\nu}_1+p \rightarrow \Lambda^0+e^+ \tag{6}$$

occur, there would be a violation of lepton conservation. While it is possible to study lepton conservation by helicity measurements, neutrino capture experiments seem to be the most direct and clean cut for such purposes.

In the rest of this Letter we shall assume that $\nu_1=\nu_2=\nu_3\equiv\nu$ and that the conservation of leptons holds.

3. Possible existence of a neutral lepton current. By using high-energy neutrinos it becomes possible to study whether reactions such as

$$\nu+p \rightarrow \nu+p \quad \text{and} \quad \nu+n \rightarrow \nu+n$$

exist or not, and if they exist, whether there is any similarity between these "neutral lepton currents" and the electromagnetic field. [See also the discussion in Sec. 8.]

4. Point structure of the lepton current. In the present theory of β decay, μ capture, etc., one assumes that all the weak reactions that contain both leptons and heavy particles can be represented by an effective Lagrangian of the type

$$-\mathfrak{L}_{\text{eff}}=\sum_{\lambda=1}^{4}[J_\lambda(x)j_\lambda(x)+J_\lambda'(x)j_\lambda'(x)], \tag{7}$$

where

$$j_\lambda(x)=-i[\psi_l^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_\nu], \tag{8}$$

$$j_\lambda'(x)=-i[\psi_\nu^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_l], \tag{9}$$

l stands for either e^- or μ^-, ψ_ν and ψ_l are the field operators for ν and l, and $J_\lambda(x)$ and $J_\lambda'(x)$ are operators that act on heavy particles (including the pions, K mesons) only. Because of the Hermiticity of $\mathfrak{L}_{\text{eff}}$, we have

$$J_\lambda'=\eta_\lambda J_\lambda^\dagger, \tag{10}$$

where

$$\eta_\lambda=+1 \quad \text{for} \quad \lambda=1,2,3,$$

$$\eta_\lambda=-1 \quad \text{for} \quad \lambda=4. \tag{11}$$

The nature of J_λ and J_λ' is known so far only in the nonrelativistic region. In the low-energy limit, the matrix elements of these heavy-particle current operators in the case of β decay are of the form

$$\langle p|J_\lambda|n\rangle=(i/\sqrt{2})u_p^\dagger\gamma_4\gamma_\lambda(G_V-G_A\gamma_5)u_n, \tag{12}$$

$$\langle n|J_\lambda'|p\rangle=(i/\sqrt{2})u_n^\dagger\gamma_4\gamma_\lambda(G_V^*-G_A^*\gamma_5)u_p, \tag{13}$$

where u_n and u_p are the spinor solutions of the free Dirac equations with the same 4-momenta as the physical neutron and proton; G_V and G_A are the Fermi and Gamow-Teller coupling constants. Due to the presence of strong interactions it is expected that in the high-energy region (12) and (13) do not hold. But one expects (7), which represents a "point interaction" for the leptons, to have a wider range of applicability.

The mere assumption that in the effective Lagrangian (7) the lepton current acts only at a single space-time point introduces rather strong restrictions on the forms of the cross sections for all neutrino and antineutrino reactions. For example, in either

$$\nu+n \rightarrow p+l^-+\text{pions} \tag{14}$$

or

$$\bar{\nu}+p \rightarrow n+l^++\text{pions}, \tag{15}$$

suppose one measures in the laboratory system the incoming momentum $\vec{k}_\nu$ and the outgoing lepton momentum $\vec{k}_l$ and does not measure the other kinematic quantities describing the reaction. The experimental cross section is then in a general case a function of the three real variables k_ν, k_l, and θ (= the angle between $\vec{k}_l$ and $\vec{k}_\nu$). Independently of the form of J_λ, assumption (7) restricts this function to a sum of three structure functions each of which has an unknown dependence on only two real variables: $E=k_\nu-k_l$ and $P=|\vec{k}_\nu-\vec{k}_l|$, which represent the energy transfer and the magnitude of the momentum transfer between the leptons and the strongly in-

teracting particles. More explicitly, assumption (7) implies[2] that the cross section for (14) [also for (15)] is of the form

$$d\sigma = dk_l\, d(\cos\theta)(4\pi k_\nu)^{-1} k_l [(k_l + k_\nu)^2 - P^2] \times [xA_+ + x^{-1}A_- + B], \quad (16)$$

where $x = (k_l + k_\nu - P)(k_l + k_\nu + P)^{-1}$ and A_+, A_-, and B are functions of E and P only.

To test the validity of (16) it is not necessary to perform a detailed experiment for specific values of E and P. One could perform a capture experiment with a neutrino beam with a known spectrum $I(k_\nu)dk_\nu$ and measure for each event the values of x, P, and E. If $N(x,P,E)dxdPdE$ is the number of events, then (16) implies that

$$I^{-1} k_\nu^{\,2} N(x,P,E) = [A_1 + A_2 x + A_3 x^2](1-x)^{-4}, \quad (17)$$

where A_i $(i = 1, 2, 3)$ are functions of P and E. Integrating (17) over P and E, one obtains

$$\sum I^{-1} k_\nu^{\,2} = (a_1 + a_2 x + a_3 x^2)(1-x)^{-4}, \quad (18)$$

where a_1, a_2, a_3 are numerical constants, and the sum extends over all events with fixed x. To test the validity of (18), less than a thousand events could be enough provided that they do not cluster around one value of x.

5. Universality of weak interactions involving $e^\pm$ and $\mu^\pm$. Neglecting the mass of the μ meson, the assumption of the universality of the weak interactions implies equal differential cross sections for μ^+ and e^+ production and for μ^- and e^- production. If the mass of the μ meson is not neglected, comparison should be made between μ and e production processes in which the energy transfer E and the magnitude P of the momentum transfer from the leptons to the strongly interacting particles are fixed, provided the point structure of the lepton current discussed in Sec. 4 is valid.

6. S-symmetry. In (16) the structure functions determined by using ν are related to the appropriate matrix elements of J_λ in Eq. (7) while those determined by using $\bar{\nu}$ are related to that of J_λ'. Thus, unless J_λ and J_λ' obey some further symmetry property the reaction rates of, e.g., (14) and (15) in general are not related to each other in any simple way. A symmetry which will be called S-symmetry is of a type so as to link J and J'.

To explain the meaning of S-symmetry we observe that by using (12) and (13) at the low-energy limit it is readily verified that J_λ and J_λ' satisfy the following relation:

$$J_\lambda' = S J_\lambda S^{-1}, \quad (19)$$

where S is the product of a 180° rotation along the y axis $[\exp(i\pi I_y)]$ in the isotopic spin space multiplied by the time-reversal operator T, i.e.,

$$S = [\exp(i\pi I_y)]T. \quad (20)$$

Condition (20) will be defined as the condition for S-symmetry.[3] If it is satisfied then there should exist identities among the matrix elements of J and J' and consequently relations between reaction rates caused by ν and $\bar{\nu}$. For example, if the mass of the lepton is neglected, the structure functions [defined in (16)] for ν and $\bar{\nu}$ processes are related to each other by

$$(A_+)_\nu = (A_-)_{\bar{\nu}}, \quad (A_-)_\nu = (A_+)_{\bar{\nu}}, \quad (B)_\nu = (B)_{\bar{\nu}}. \quad (21)$$

Experimentally it is not known at present whether S-symmetry is satisfied at energies of the order of 100 Mev and up. Theoretically it has been customary[4] to assume bare particle universal Fermi interactions for all Fermions. If such an assumption is made, S-symmetry follows.

7. Conserved vector current and proportionality with the electromagnetic current. The heavy-particle current J_λ can be written as a sum,

$$J_\lambda = V_\lambda + A_\lambda, \quad (22)$$

where V_λ and A_λ are, respectively, its vector part and axial-vector part. Recently, Feynman and Gell-Mann[5] proposed that the vector part V_λ satisfies the conservation law

$$\partial V_\lambda / \partial x_\lambda = 0. \quad (23)$$

Furthermore it is proposed that V_λ is equal to the corresponding isotopic vector part of the electromagnetic current times a constant (hereafter called the proportional vector current hypothesis). A sensitive test of the proposal can be given by studying reactions such as

$$\nu + n \to e^- + p, \quad (24)$$

$$\bar{\nu} + p \to e^+ + n, \quad (25)$$

and comparing them with existing data on electron scattering by nucleons[6] at the same momen-

tum transfer to the nucleons.

Making the proportional vector current hypothesis, the electron scattering experiments[6] show that V_λ is proportional to

$$F(q^2)=[1+\frac{1}{12}q^2\alpha^2]^{-2}, \quad \alpha=0.8\times10^{-13}\ \text{cm},$$

where q^2 is the invariant four-momentum transfer squared. We determine the constant of proportionality by the condition $G_V=10^{-5}/M^2$. For orientation purposes we assumed that A_λ is given by

$$\langle p|A_\lambda|n\rangle=1.2\ G_V F(i/\sqrt{2})u_p^\dagger\gamma_4\gamma_\lambda\gamma_5 u_n,$$

and calculated the cross sections for (24) and (25). The results are exhibited in Fig. 1.

8. Possible existence of a weakly coupled Boson $W^\pm$. The question whether the weak interactions are "transmitted" by a Boson field was already discussed in Yukawa's original work on the meson. If such a Boson field $W^\pm$ exists, it must have spin 1, and one can also conclude that its mass m_W is $\gtrsim m_K$. The nonlocality of the weak interactions implied by the finite mass of the transmitting field W has been discussed before.[7] For μ decay, the change in the Michel parameter ρ is given by

$$\rho-0.75\cong\tfrac{1}{3}(m_\mu/m_W)^2.$$

A value of $m_W>m_K$ is thus consistent with existing experiments.

The coupling of the $W^\pm$ to the leptons is characterized by a coupling constant,

$$g^2/4\pi=(\pi\sqrt{2})^{-1}G_V m_W^2<6.4\times10^{-7}.$$

The decay rate of $W\to e+\nu$ is found to be

$$\lambda_{W\to e+\nu}=G_V m_W^3(6\pi\sqrt{2})^{-1}, \tag{26}$$

and the ratio of the decay rate of $W\to\mu+\nu$ to that of $W\to e+\nu$ is

$$2v_\mu^2(3+v_\mu)/(1+v_\mu)^3, \tag{27}$$

where v_μ is the velocity of the muon in the rest system of W. Decays of W into pions are also possible. The lifetime of W is $<10^{-17}$ sec.

While the existence of a $W^\pm$ as a virtual particle does not change in any essential way the considerations of the previous sections, its production makes possible a much higher cross section for neutrino reactions through the pair creation of W^+ and l^- in the Coulomb field of a target nucleus:

$$\nu+Z\to W^++l^-+Z. \tag{28}$$

The cross section for (28) is large compared with those without W production. At values of $k_\nu\gg m_W^2/2q_0\approx2$ Bev, where $q_0=\hbar/$nucleon radius, the cross section is given by

$$\sigma\cong(6\pi\sqrt{2})^{-1}(137)^{-2}Z^2G_V[\ln(2k_\nu q_0/m_W^2)]^3, \tag{29}$$

which, for $Z=26$, is of the order of 10^{-35} cm^2. The $W^\pm$ produced are easily identifiable through their decay products $e+\nu$ or $\mu+\nu$. If experimentally no $W^\pm$ is found, it would be possible to set a lower limit on the value of m_W.

The existence of the intermediate Boson $W^\pm$ has[8] been discussed in connection with the question of the absence of $\mu^\pm\to e^\pm+\gamma$. On reasonably sure grounds one could conclude that the existence of $W^\pm$ requires that in the notation of (1), (2), (3), $\nu_1\neq\nu_2$. In other words, the existence of W would imply that reaction (4) does not occur.

For processes without W production, the discussions of Sec. 4 above concerning a point structure of the lepton current remain unchanged, but the current J_λ now includes the effect of the propagator of W. Furthermore a corresponding change in the proportional vector current hypothesis is necessary.

The question of a neutral W^0 will not be examined here.

9. Interactions with extremely large momentum transfers. For momentum transfers of the order of $(G_V)^{-1/2}\sim300$ Bev/c, quantities other

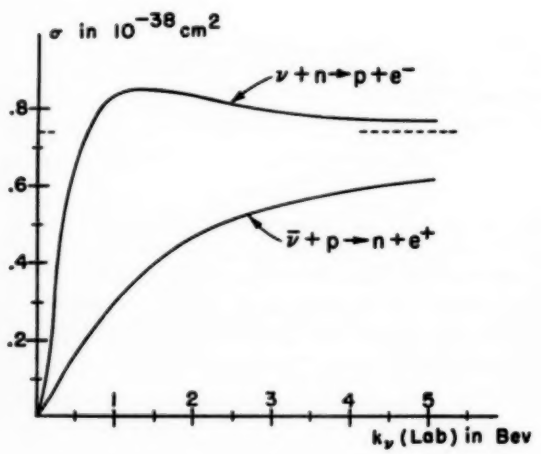

FIG. 1. "Elastic" neutrino cross sections. The dashed line represents the limit of σ as $k_\nu\to\infty$.

than the lowest power of G_V become important. For example, for $e^- + \nu \rightarrow \mu^- + \nu$, the cross section predicted by the lowest perturbation formula approaches the limit set by the unitary condition as the neutrino momentum in the center-of-mass system, $p_\nu \sim 300$ Bev. For processes involving pion clouds, high-order terms in G_V presumably also become important as $p_\nu \sim 300$ Bev. The description of the weak interaction at such high energies would seem to need ideas radically different from the current picture.

We wish to thank Professor M. Schwartz for many discussions.

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[2]In Eq. (16), we neglect the mass of the lepton. For reactions in which the mass of the lepton may not be neglected, equations similar to but slightly more complicated than (16) result. For reactions in which strange particles are produced, (16) remains valid.

[3]By using the *CPT* theorem it is easy to see that the S-symmetry is closely related to the classification of weak interactions given by S. Weinberg, Phys. Rev. 112, 1375 (1958).

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ENHANCEMENT OF BREMSSTRAHLUNG PRODUCED BY 575-Mev ELECTRONS IN A SINGLE CRYSTAL OF SILICON*

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Experiments have been performed[1-3] to observe the expected enhancement of the bremsstrahlung produced by high-energy electrons when they pass through a single crystal.[4] The theory of Überall has been critically examined, and improved calculations have been made by Schiff.[5] The purpose of this report is to present the preliminary results of the revised experiment performed at Stanford, using a more sensitive technique of detection of the enhancement.

A single crystal of silicon, 0.013 in. thick and cut along the [110] plane, was aligned normal to the x-ray beam as defined by a 0.025-in. collimator, by the Laue back-reflection method. The crystal in its holder was mounted in a remote-controlled double-goniometer in which it could be rotated accurately about two coplanar perpendicular axes normal to the electron beam of the accelerator. The normal position of the [110] plane of the crystal with respect to the electron beam was established by using optical methods to transfer the x-ray alignment. Electrons of 575 Mev produced bremsstrahlung in the single crystal and were then deflected. The deflected electron beam was monitored by a standard secondary emission monitor.[6] The bremsstrahlung was monitored by a double-ionization chamber consisting of two identical ionization chambers constructed in one envelope, each preceded by appropriate beryllium converters. This unit was mounted accurately so that the axis of both the ionization chambers was in line with the incident beam direction. Continuous flow of purified hydrogen was maintained in the chamber at a constant pressure. The sensitivity of the first ionization chamber was a maximum around a photon energy of 30 Mev. The second ionization chamber looked at pairs which on the average have penetrated a larger range of beryllium and are therefore produced by a relatively harder component of the bremsstrahlung. Since the enhancement is large in the soft component of the bremsstrahlung, the relative increase of the charge in the first ionization chamber over that in the second ionization chamber would be quite pronounced. The charges collected in the two ionization chambers were fed separately to two dc amplifiers, the output of which was balanced using two potentiometers. The increase of the measured ratio as the crystal plane approached the correct angle with respect to the electron beam, would indicate the enhancement

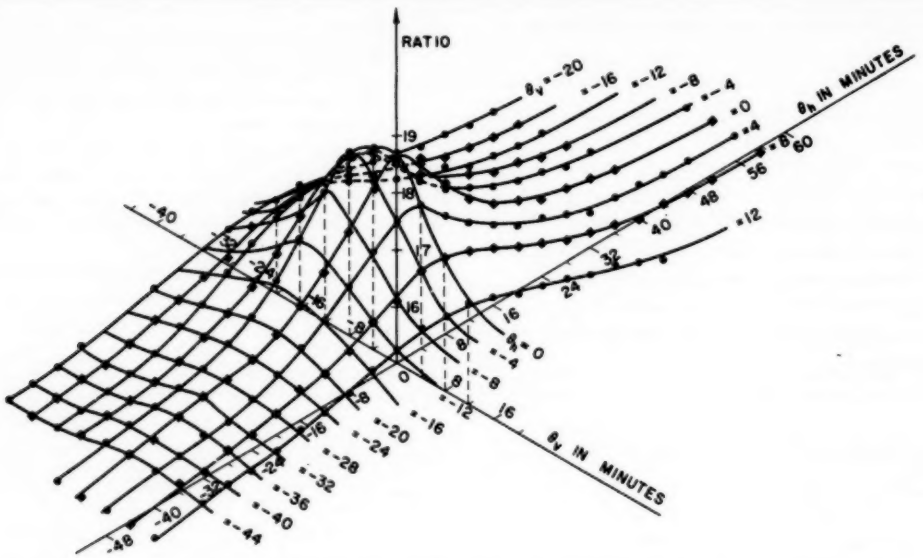

FIG. 1. The ratio of the charges, a measure of the enhancement of the soft component of the bremsstrahlung, is plotted against the angle between the direction of the incident electron beam and the [110] axis of the Si crystal, in which the bremsstrahlung is produced, about the horizontal and vertical axes of rotation. θ_h is the angle of rotation of the crystal about the horizontal axis, and θ_v is the angle of rotation about the vertical axis.

of the soft component of the bremsstrahlung. Thus, this method provides a more sensitive way to detecting the enhancement than that used previously,[1] using the photoproduction of mesons.

The ratio of the charges, a measure of the enhancement, was observed for various angles as the crystal was rocked back and forth about the vertical and horizontal axes. An angular grid of about $\pm 16'$ of θ_v, the rotation about the vertical axis, and about $\pm 48'$ of θ_h, the rotation about the horizontal axis, was covered in steps of $4'$. The data are shown in Fig. 1 in a three-dimensional plot of the ratio vs θ_v and θ_h. It is clear that the soft component of the bremsstrahlung is definitely enhanced. The theoretically predicted dip in the center of the peak could not be observed. Calculations are in progress to estimate quantitatively the agreement between the theory and the experimental data; these calculations depend in detail on the precise energy response curve of the instrument to photons of various energies.

I am happy to acknowledge the help of Professor W. K. H. Panofsky, Professor H. C. DeStaebler, and Professor L. I. Schiff, and of other colleagues in this laboratory.

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PION FORM FACTORS FROM POSSIBLE HIGH-ENERGY ELECTRON-POSITRON EXPERIMENTS

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Theoretical work on nucleon structure[1,2] has revealed the important role of the photon-(n-pion) vertices in the theory of the nucleon form factors. In particular the $\gamma - 2\pi$ vertex is thought to be the most important one contributing to the isotopic vector part of the structure, and the $\gamma - 3\pi$ vertex the most important one for the isotopic scalar part. Recent technical developments showing the feasibility of colliding beam experiments[3] make it appealing to think of possible direct measurements of the photon-pion vertices through processes of the sort

$$e^+ + e^- \to n \text{ pions}. \qquad (1)$$

We shall here discuss the reactions (1) in their lowest electromagnetic approximation. Examination of the higher order terms in the electromagnetic coupling constant may become necessary when detailed experiments are carried out.

Let us consider the reaction (1) in the center-of-mass frame. In the lowest electromagnetic approximation only the $^3S_1$ and $^3D_1$ states for the initial positron-electron state will contribute to (1) and the final n-pion state must have parity minus, charge-conjugation quantum number minus, total angular momentum one, and total isotopic spin one if n is even, zero if n is odd. In particular the process is forbidden in this order if the final pions are all neutrals. The S-matrix element for (1) is given by

$$S_{fi} = \delta(p^{(1)} + p^{(2)} + \cdots + p^{(n)} - e^{(+)} - e^{(-)}) \times \frac{2\pi e}{K^2}(\bar{v}(e^+)\gamma_\nu u(e^-))\langle p^{(1)}, p^{(2)}, \cdots p^{(n)}|j_\nu(0)|0\rangle, \qquad (2)$$

where $p^{(1)}, p^{(2)}, \cdots p^{(n)}$ are the final pion momentum four-vectors, $e^{(+)}$ and $e^{(-)}$ the positron and electron momenta, respectively, $K = e^{(+)} + e^{(-)}$, $\bar{v}(e^+)$ and $u(e^-)$ are the Dirac spinors, and the matrix element of the electric current operator $j_\nu(x)$ is taken between the vacuum state and the final state of n outgoing pions.

We define

$$J_\nu(p^{(1)}, p^{(2)}, \cdots p^{(n)}) = (2\pi)^{3n/2}\langle p^{(1)}, p^{(2)}, \cdots p^{(n)}|j_\nu(0)|0\rangle. \qquad (3)$$

It follows from gauge invariance that, in the center-of-mass system for (1), J_ν has only the space component $\vec{J}$. From parity conservation $\vec{J}$ is a polar vector formed from the n vectors $\vec{p}^{(1)}, \vec{p}^{(2)}, \cdots \vec{p}^{(n)}$ if n is even, an axial vector formed from these vectors if n is odd. Inserting (3) into (2) one finds for the total cross section

$$\sigma = \frac{\alpha}{32E^4(2\pi)^{3n-5}}\int d\vec{p}^{(1)} d\vec{p}^{(2)} \cdots d\vec{p}^{(n)} \delta(\omega^{(1)} + \omega^{(2)} + \cdots + \omega^{(n)} - 2E)\delta(\vec{p}^{(1)} + \vec{p}^{(2)} + \cdots \vec{p}^{(n)})|\vec{J}|^2 \sin^2\theta, \qquad (4)$$

where α is the fine structure constant (another factor α is contained in $|\vec{J}|^2$), E is the electron (or positron) energy, $\omega^{(i)}$ is the energy of the ith pion, and θ is the angle between the vector $\vec{J}$ and the line of collision. The dependence on θ, as $\sin^2\theta$, is therefore a direct consequence of gauge invariance independent of any knowledge of the γ-pion vertices. In the case of two final pions, with only one vector available, $\vec{J}$ must be proportional to $\vec{p}^{(1)} = -\vec{p}^{(2)}$ and the angular distribution is thus completely determined of the form $\sin^2\theta$. In the case of three final pions, $\vec{J}$ must be proportional to the only available axial vector $\vec{p}^{(1)} \times \vec{p}^{(2)} = -\vec{p}^{(1)} \times \vec{p}^{(3)}$, etc, and therefore the distribution of the angle θ between the normal to the plane in which the three pions are produced and the line of collision is uniquely given by $\sin^2\theta$.

The final state of two pions (necessarily one positive, one negative) produced according to (1) is necessarily a p state of relative orbital angu-

lar momentum. The matrix element (3) for two pions is expressed in terms of the pion factor $M(K^2)$ by[2]

$$J_\nu(p^{(1)},p^{(2)}) = e(4\omega^{(1)}\omega^{(2)})^{-1/2}M(K^2)(p^{(1)}-p^{(2)})_\nu. \quad (5)$$

The differential cross section, always in the center-of-mass frame, is given by

$$d\sigma = \frac{2\pi\alpha^2}{32E^5}|M(-4E^2)|^2p^3\sin^2\theta d(\cos\theta), \quad (6)$$

where $p = |\vec{p}^{(1)}| = |\vec{p}^{(2)}|$. Frazer and Fulco[4] propose a resonant form of $M(K^2)$ as a simplest explanation of the isotopic vector part of the nucleon structure. According to (6) one finds for $E = 230$ Mev, for which $M(-4E^2)$ comes close to its maximum, a total cross section for $e^+ + e^- \to \pi^+ + \pi^-$ of 4.6×10^{-31} cm^2, which is $\cong 17$ times bigger than the value one would get from perturbation theory ($M = 1$). The cross section then falls down very rapidly at higher energies.

The final state of three pions (necessarily one positive, one negative, one neutral) produced according to (1) is a superposition of states with $l = L = 1$, $l = L = 3$, $l = L = 5$, etc., where l is the relative $\pi^+\pi^-$ angular momentum and L the angular momentum of the π^0 relative to $\pi^+\pi^-$. The matrix element (3) for three pions can be expressed in terms of a form factor H^* depending on three independent scalars which can be chosen as E, ω_+ (the energy of the positive pion), and ω_- (the energy of the negative pion),[2]

$$J_\nu(\vec{p}^{(+)},\vec{p}^{(-)},\vec{p}^{(0)}) = -i(8\omega_+\omega_-\omega_0)^{-1/2}H^*\epsilon^{\nu\rho\sigma\tau}p_\rho^{(+)}p_\sigma^{(-)}p_\tau^{(0)}. \quad (7)$$

The vector $\vec{J}$ is then given by

$$\vec{J}(\vec{p}^{(+)},\vec{p}^{(-)},\vec{p}^{(0)}) = 2(8\omega_+\omega_-\omega_0)^{-1/2}H^*E(\vec{p}^{(+)}\times\vec{p}^{(-)}). \quad (8)$$

With our choice of the independent kinematical parameters the differential cross section can be written as

$$\frac{d^2\sigma}{d\omega_+ d\omega_-} = \frac{\alpha}{(2\pi)^2}\frac{|H|^2}{64E^2}(\vec{p}^{(+)}\times\vec{p}^{(-)})^2\sin^2\theta d(\cos\theta). \quad (9)$$

Measurement of this cross section will thus inform us directly about $|H|^2$ which plays a relevant role in the nucleon structure problem. At present there is no knowledge at all about this form factor. Even a rough knowledge of its average magnitude would be very important for our understanding of the nucleon structure. Bosco and De Alfaro have assumed a constant approximation for H, $H = (\lambda e)/\mu^3$, and they claim that double pion photoproduction puts an upper limit $\cong 6$ for λ.[5] At a given E the maximum of $d\sigma$ is obtained for those configurations of the three final pions for which the area of the triangle formed with their momenta is maximum. For such configurations and for $E \cong 500$ Mev, we find, using a constant H and the above value for λ, $d^2\sigma/d\omega_+ d\omega_- \cong 1.3\times10^{-35}$ cm^2 Mev^{-2}.

We are indebted to Professor W. K. H. Panofsky for a stimulating seminar on the possibilities of colliding-beam experiments.

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EFFECTS OF PION-PION INTERACTION IN ELECTROMAGNETIC PROCESSES

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Resonances in pion-pion scattering have been proposed by some authors.[1] Recently Frazer and Fulco,[2] following the approach of Chew and Mandelstam, and assuming a resonance in the $J=1$, $I=1$ state of two pions, derived an electromagnetic form factor for the pion which has a resonance-like behavior as a function of the invariant momentum transfer. They show that the position and width of the resonance can be adjusted to give a good fit to the isotopic vector part of the nucleon electromagnetic structure.

We have calculated the effect of this pion form factor on the photon propagator, and have obtained corrections of order e^2 to several purely electromagnetic processes. In spite of the large mass of the intermediate pion pair, these corrections turn out to be perhaps detectable in electron scattering experiments at center-of-mass energies smaller than those expected from machines now under construction.[3] Such experiments would have the obvious advantage of giving information on the pion form factor independently of the complications of nuclear structure.

Our method of calculation parallels that used by Källén in his discussion of charge renormalization. Our modifications consist of using intermediate bosons instead of fermions and of inserting the pion form factor.

We will outline here only the scheme of calculation, reserving the details and further results for publication elsewhere. We write first, following Källén, the renormalized photon propagator in the momentum representation:

$$D_{\mu\nu}^{F'}(p)=(p^2-i\epsilon)^{-1}\{\delta_{\mu\nu}+(\delta_{\mu\nu}-p_\mu p_\nu/p^2)\times[R(p^2)+iI(p^2)]\}, \quad (1)$$

with

$$R(p^2)=\overline{\Pi}(0)-\overline{\Pi}(p^2), \quad (2a)$$

$$I(p^2)=-\pi\Pi(p^2). \quad (2b)$$

The imaginary part $I(p^2)$, which involves the product of two current operators, is expressed as a sum over physical intermediate states through[4]

$$\Pi(p^2)=-(3p^2)^{-1}\sum_{p^{(z)}=p}\langle 0|j_\mu|z\rangle\langle z|j_\mu|0\rangle. \quad (3)$$

Restricting the sum to intermediate states consisting of two charged physical pions, we multiply the usual first-order matrix element by the pion form factor[5] $F_\pi(p^2)$ and carry out the sum. We then construct the real part

$$R(p^2)=P\int_0^\infty \frac{p^2\Pi(-a)}{a(p^2+a)}da, \quad (4)$$

where P designates the principal value.

As the form factor of FF has a rather complicated form for analytical work, we use a simple analytic fit:

$$|F_\pi(p^2)|^2=|F_\pi'(p^2)|^2, \quad 0>p^2>p_0^2$$

$$=1, \quad p_0^2>p^2 \quad (5)$$

where

$$|F_\pi'(p^2)|^2=A[(p^2+C)^2+B^2]^{-1},$$

$$|F_\pi'(0)|^2=|F_\pi'(p_0^2)|^2=1. \quad (6)$$

The numerical results which we give below for electron scattering are based on the choice

$$A=66.25, \quad B=1.5, \quad C=8, \quad (7)$$

which corresponds to the FF form factor having a peak at $p^2=-8$, using always units of the pion mass. (The other FF form factors give slightly smaller corrections.)

The final result is

$$\Pi(p^2)=(\alpha/6\pi)z^{3/2}\theta(-p^2-4)|F_\pi(p^2)|^2, \quad (8)$$

$$R(p^2)=(\alpha/6\pi)\{[1-|F_\pi'(p^2)|^2]z_0^{3/2}f(z/z_0)-f(z)+p^2|F_\pi'(p^2)|^2[-\gamma+\delta(C+p^2)/B]\}, \quad (9)$$

where

$$\alpha \approx 1/137,$$

$$z = 1 + 4/p^2, \quad z_0 = 1 + 4/p_0^2,$$

$$\theta(x > 0) = 1, \quad \theta(x < 0) = 0,$$

$$f(z) = \tfrac{2}{3} + 2z + z^{3/2}\ln\left(\frac{z^{1/2} - 1}{z^{1/2} + 1}\right). \tag{10}$$

For real positive z, the absolute value of the argument of the logarithm must be taken in (10). For negative z, one must replace $z^{1/2}\ln[(z^{1/2} - 1)/(z^{1/2} + 1)]$ by $-2y$ arc coty, with $y = (-z)^{1/2}$. For complex values of z, $f(z)$ is defined in a plane cut in the negative real axis. The constants γ and δ are defined by[6]

$$\gamma + i\delta = \tfrac{1}{4}(1 - z')z_0^{3/2} f(z'/z_0) \tag{11}$$

with

$$z' = 1 - 4/(C + iB).$$

We have applied this result to evaluate the correction to Møller and Bhabha scattering of electrons due to pion vacuum polarization. The fractional correction $k(E, \cos\theta)$, where E is the center-of-mass energy of each electron and θ the c.m. angle of scattering, takes the form in the extreme relativistic case:

$$\text{Møller:} \quad k = (1 + X)R(p_1^2) + (1 - X)R(p_2^2), \tag{12}$$

$$\text{Bhabha:} \quad k = (1 + Y)R(p_3^2) + (1 - Y)R(p_4^2), \tag{13}$$

with

$$X = 4\cos\theta(3 + \cos^2\theta)^{-1}, \tag{14}$$

$$Y = (1 + \cos\theta)(3 - \cos\theta)(3 + \cos^2\theta)^{-1}, \tag{15}$$

and with

$$p_1^2 = p_3^2 = 2E^2(1 - \cos\theta),$$

$$p_2^2 = 2E^2(1 + \cos\theta),$$

$$p_4^2 = -4E^2. \tag{16}$$

In Figs. 1 and 2 we have plotted some examples of these results.

The largest corrections are obtained for large-angle Bhabha scattering, which is dominated by the timelike momentum transfer p_4^2. For this reason the large-angle plot in Fig. 2 reproduces essentially the correction to the photon propagator for timelike p^2. From an experimental curve showing this behavior could be deduced unambiguously the position and width of the resonant part of the pion form factor, free of the uncertain interpretations of nuclear structure.

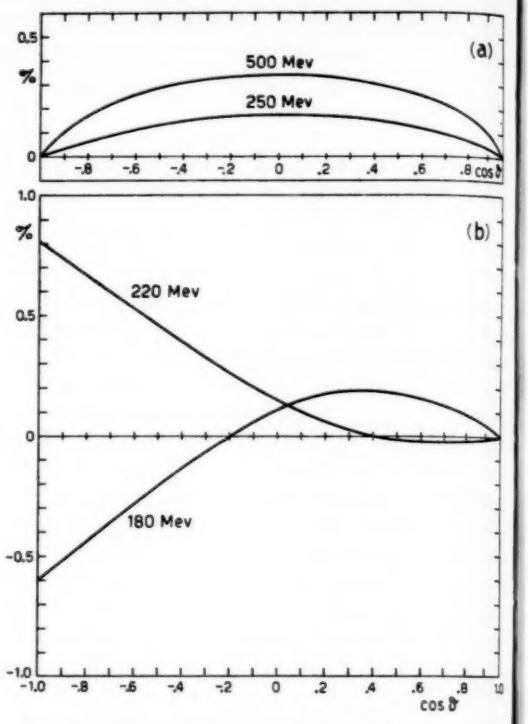

FIG. 1. Percentage correction as a function of c.m. angle for fixed c.m. energy of each electron: (a) Møller scattering and (b) Bhabha scattering.

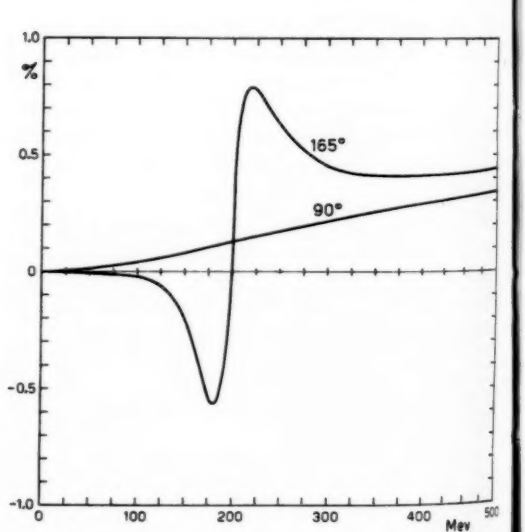

FIG. 2. Percentage correction to Bhabha scattering as a function of c.m. energy of each electron for fixed c.m. angle.

Of course annihilation processes leading to muon pairs or to two or more pions will become of interest at these energies. For the muon case we get a small correction similar to that for large-angle Bhabha scattering. For the very important case of annihilation into pion pairs one is studying directly the pion form factor. For example, at 90° in the c.m. system one gets

$$\frac{d\sigma(\pi\pi)}{d\sigma(\text{Bhabha})} = \frac{1}{16}\left(\frac{\vec{p}^2}{E^2}\right)^{3/2} |F_\pi(-4E^2)|^2, \qquad (17)$$

where $\vec{p}$ and E are the momentum and energy of each pion. This is certainly, therefore, the best means of investigating the pion form factor. If a resonant pion form factor is found in this way, then the small effects we have calculated become of relevance in the interpretation of scattering experiments in terms of a possible failure of quantum electrodynamics at small distances.[7]

However, while the positron-electron experiment appear to be the most promising for elucidating the pion form factor, electron-electron experiments will probably be done first and the above corrections for Møller scattering may be of interest. We are investigating other electromagnetic processes in which pion-pion interaction plays a role.

It is a pleasure to thank Professor M. Cini for encouragement and advice.

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[2]W. R. Frazer and J. R. Fulco, Phys. Rev. Letters 2, 365 (1959) and University of California Radiation Laboratory Report UCRL-8880 (unpublished), hereafter referred to as FF.

[3]Barber, Richter, Panofsky, O'Neill, and Gittelman, Stanford University, High-Energy Physics Laboratory Report HEPL-170, June, 1959 (unpublished).

[4]G. Källén, Handbuch der Physik (Springer-Verlag, Berlin, 1958), Vol. V-1, Sec. 43.

[5]G. F. Chew, R. Karplus, S. Gasiorowicz, and F. Zachariasen, Phys. Rev. 110, 265 (1958).

[6]For the A, B, and C given in Eq. (7), we get $\gamma = 0.047$, $\delta = 0.106$.

[7]It must be emphasized that the usual electromagnetic radiative corrections, not considered here, are large and must be included.

INVARIANT COMMUTATORS FOR THE QUANTIZED GRAVITATIONAL FIELD*

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(Received February 8, 1960)

The problem of developing a formalism within which a union of the general theory of relativity with the quantum theory might successfully be brought about has been attacked by many authors.[1] The difficulties encountered in this work have seemed to be twofold: (1) the nonlinearity of the general theory, and (2) its coordinate invariance, which leads to constraints on the Cauchy data for the dynamical equations. The real difficulty may lie elsewhere, however. Approaches to the problem have almost without exception been made via a Hamiltonian or quasi-Hamiltonian canonical formalism, with an attendant loss of manifest covariance. It is the purpose of this Letter to raise the question of the suitability of canonical procedures and to suggest, by obtaining, without their aid, an explicit covariant expression for the commutators of the theory, that it may be possible to avoid them entirely.

We begin by pointing out that, apart from problems of factor ordering, the nonlinearity of the dynamical equations offers no complications for the commutators, contrary to a widespread impression. As Peierls[2] has shown, the commutator of two dynamical variables is determined by the variation in one due to an infinitesimal change in the action proportional to the other. But infinitesimal variations are propagated by means of linear equations, and standard techniques are available for handling the associated propagation functions.

Secondly, we note that in the general theory of relativity, well-defined commutators can be obtained only between absolute invariants. The infinitesimal coordinate transformation law,

$$\delta g_{\mu\nu} = \xi_{\mu\cdot\nu} + \xi_{\nu\cdot\mu}, \quad (\xi_\mu \text{ infinitesimal}) \qquad (1)$$

leads to the characterization

$$(\delta A/\delta g_{\mu\nu})_{\cdot\nu} = 0, \quad (\delta B/\delta g_{\mu\nu})_{\cdot\nu} = 0, \tag{2}$$

of any two absolute invariants A and B, which are constructed solely out of the metric field $g_{\mu\nu}$.[3] Equations (2) may hold either as identities or in virtue of the field equations.

Consider now the following change in the action:

$$S \to S + \epsilon B, \tag{3}$$

where ϵ is an infinitesimal constant. Such an alteration will produce a change in the invariant A, the precise nature of which depends upon the boundary conditions selected. For example, we may adopt advanced or retarded boundary conditions, in which the dynamical states of the system before and after the alteration (3) are taken to coincide in the remote future or remote past, respectively.[4] Denoting the corresponding changes in A by δ^+A and δ^-A, respectively, we may express the commutator of A and B in the form

$$\delta^+A - \delta^-A = -ie[A, B]. \tag{4}$$

(We choose units in which $\hbar = c = 16\pi G = 1$.) That Eq. (4) follows, in the case of normal canonical systems, from the usual canonical definition of the commutator (Poisson bracket) has been proved by Peierls.[2] In the present case we shall adopt Eq. (4) itself as the definition of the commutator, the utility of this procedure resting on the fact that an independent evaluation of δ^+A-δ^-A is possible.

Ignoring, for the present, problems of factor ordering, we begin by writing

$$\delta^{\pm}A = \int (\delta A/\delta g_{\mu\nu})\delta^{\pm}g_{\mu\nu}\, d^4x. \tag{5}$$

Since $g_{\mu\nu}$ is not an invariant its variations $\delta^{\pm}g_{\mu\nu}$ are determined only up to a coordinate transformation (1). In virtue of (2), however, the variations $\delta^{\pm}A$ remain well defined. The differential equations satisfied by the $\delta^{\pm}g_{\mu\nu}$ are obtained by varying the original field equations,

$$0 = \delta S/\delta g_{\mu\nu} = G^{\mu\nu} \equiv g^{1/2}(R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}R). \tag{6}$$

(Here g is the negative of the determinant of $g_{\mu\nu}$ and $R^{\mu\nu}$ is the Ricci tensor.) Under the alteration (3) the new field equations lead to

$$\delta^{\pm}G^{\mu\nu} = -\epsilon\delta B/\delta g_{\mu\nu}. \tag{7}$$

It is not hard to show that when the field equations (6) are satisfied, the left side of (7) becomes[5]

$$\delta^{\pm}G^{\mu\nu} = \tfrac{1}{2}g^{1/2}(g^{\mu\sigma}g^{\nu\tau} - \tfrac{1}{2}g^{\mu\nu}g^{\sigma\tau})g^{\rho\lambda}(\delta^{\pm}g_{\sigma\tau\cdot\rho\lambda} + \delta^{\pm}g_{\rho\lambda\cdot\sigma\tau} - \delta^{\pm}g_{\sigma\rho\cdot\tau\lambda} - \delta^{\pm}g_{\tau\lambda\cdot\sigma\rho}). \tag{8}$$

One may then easily verify that the variation (1), if substituted into (8), gives zero when (6) is satisfied.[6] The general solutions of Eqs. (7) are therefore obtained by adding (1) to particular solutions determined by appropriate boundary and supplementary conditions.

As supplementary conditions it is convenient to choose

$$(g^{\mu\sigma}g^{\nu\tau} - \tfrac{1}{2}g^{\mu\nu}g^{\sigma\tau})\delta^{\pm}g_{\sigma\tau\cdot\nu} = 0. \tag{9}$$

Equations (7) then take the form

$$g^{1/2}(g^{\mu\sigma}g^{\nu\tau} - \tfrac{1}{2}g^{\mu\nu}g^{\sigma\tau})g^{\rho\lambda}\delta^{\pm}g_{\sigma\tau\cdot\rho\lambda} - 2g^{1/2}R^{\mu\sigma\nu\tau}\delta^{\pm}g_{\sigma\tau} = -2\epsilon\delta B/\delta g_{\mu\nu}, \tag{10}$$

of which the solutions are

$$\delta^{\pm}g_{\mu\nu} = \epsilon\int G^{\pm}{}_{\mu\nu\alpha\beta}(x,z)\frac{\delta B}{\delta g_{\alpha\beta}}\, d^4z, \tag{11}$$

where $G^{\pm}{}_{\mu\nu\alpha\beta}$ are the advanced and retarded Green's functions for Eq. (10), satisfying

$$g^{\sigma\tau}G^{\pm}{}_{\mu\nu\alpha\beta\cdot\sigma\tau} - 2R_{\mu\ \nu\ }{}^{\sigma\ \tau}G^{\pm}{}_{\sigma\tau\alpha\beta} = -(\bar{g}_{\mu\alpha}\bar{g}_{\nu\beta} + \bar{g}_{\mu\beta}\bar{g}_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta})g^{-1/4}(x)\delta^{(4)}(x,z)g^{-1/4}(z). \tag{12}$$

Green's functions of this type have been studied by Brehme and the author,[7] who have called them bi-tensors on account of their transformation properties at two different space-time points. The notation of reference 7 is followed here. Indices from the first part of the Greek alphabet refer to the point z and those from the latter part to the point x. The quantity $\bar{g}_{\mu\alpha}$ is the bi-vector which effects parallel displacements along the geodesic between x and z. The 4-dimensional delta function is regarded as a scalar density of weight $\frac{1}{2}$ at both x and z. The general nature of the propagation characterized by the above Green's functions has been described by Hadamard.[8] Explicit forms for such Green's functions are given in reference 7. Typically, they are singular on the light cone

(delta function of the square of the geodetic interval), nonvanishing but finite inside, and vanishing outside.

It is now important to check that the solutions (11) satisfy the supplementary conditions (9) which were invoked in order to get them in the first place. By using the techniques of reference 7, it may be shown that

$$(g^{\mu\sigma}g^{\nu\tau} - \tfrac{1}{2}g^{\mu\nu}g^{\sigma\tau})G^{\pm}_{\sigma\tau\alpha\beta\cdot\nu} = -G^{\pm\mu}_{\alpha\cdot\beta} - G^{\pm\mu}_{\beta\cdot\alpha}, \quad (13)$$

where $G^{\pm\mu}_{\alpha}$ are the advanced and retarded bivector Green's functions satisfying

$$g^{\nu\sigma}G^{\pm\mu}_{\alpha\cdot\nu\sigma} = -\bar{g}^{\mu}_{\alpha}g^{-1/4}(x)\delta^{(4)}(x,z)g^{-1/4}(z). \quad (14)$$

From (11) and (13) it then follows that

$$(g^{\mu\sigma}g^{\nu\tau} - \tfrac{1}{2}g^{\mu\nu}g^{\sigma\tau})\delta^{\pm}g_{\sigma\tau\cdot\nu} = -2\epsilon\int G^{\pm\mu}_{\alpha\cdot\beta}(x,z)\frac{\delta B}{\delta g_{\alpha\beta}}d^4z, \quad (15)$$

which, after an integration by parts and use of Eq. (2), leads back to (9), showing the complete self-consistency of these supplementary conditions.

Finally, therefore, upon substituting (11) into (5) and making use of (4), we obtain, for the commutator,

$$[A,B] = i\int d^4x\int d^4z\,\frac{\delta A}{\delta g_{\mu\nu}}G_{\mu\nu\alpha\beta}(x,z)\frac{\delta B}{\delta g_{\alpha\beta}}, \quad (16)$$

where

$$G_{\mu\nu\alpha\beta} \equiv G^{+}_{\mu\nu\alpha\beta} - G^{-}_{\mu\nu\alpha\beta}. \quad (17)$$

To the extent that it is possible to ignore the noncommutativity of factors on the right-hand side of (16), this commutator can be shown to satisfy all the usual identities. Antisymmetry follows immediately from the reciprocity relation satisfied by the propagation function[9]:

$$G_{\mu\nu\alpha\beta}(x,z) \equiv -G_{\alpha\beta\mu\nu}(z,x). \quad (18)$$

The identity,

$$[A,BC] \equiv [A,B]C + B[A,C], \quad (19)$$

is straightforward. The proof of the Jacobi identity, on the other hand, is quite lengthy since it involves variational derivatives of the propagation function. These derivatives can be computed by varying the defining equations (12); the proof then carries through, after a considerable amount of labor.

The first step in attempting to go beyond these results by taking into account the noncommutativity of factors consists of establishing an algorithm for the evaluation of the right-hand side of Eq. (16). One must require that the propagation function $G_{\mu\nu\alpha\beta}$ is first to be inserted as a replacement for $\delta g_{\alpha\beta}$ in all the places in which it occurs in the variation δB, and that the resulting "product" is then inserted as a replacement for $\delta g_{\mu\nu}$ in the variation δA, or, alternatively, that the process of insertion is first performed in δA and then in δB. That the two procedures are equivalent is evident from the familiar properties of commutator brackets when one takes note of the fact that in computing the commutator of two absolute invariants one may work directly with the factors (the metric tensor and its derivatives) out of which they are built, proceeding as if the metric tensor had the simple commutation rule,

$$[g_{\mu\nu}, g_{\alpha\beta}] = iG_{\mu\nu\alpha\beta}. \quad (20)$$

It is clear from these considerations that the question of the consistency of a quantum theory of gravitation falls squarely on the propagation function, which, contrary to the case of linear field theories, is here an operator. The problem of a consistent definition of this "operator-propagator" is under active consideration at the present time.

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*Work supported by the U. S. Air Force Office of Scientific Research.

[1]P. A. M. Dirac, Proc. Roy. Soc. (London) A246, 333 (1958); Phys. Rev. 114, 924 (1959). S. Deser and R. Arnowitt, Phys. Rev. 113, 745 (1959); also subsequent papers to appear in collaboration with C. W. Misner. For references to earlier work see P. G. Bergmann, Suppl. Helv. Phys. Acta, IV, 79 (1956).

[2]R. E. Peierls, Proc. Roy. Soc. (London) A214, 143 (1952).

[3]Covariant differentiation is here indicated by the dot followed by an index. The notation for variational differentiation appearing in Eq. (2) is the standard adopted for most work in quantum field theory. [See, for example, N. N. Bogoliubov and D. V. Shirkov, Introduction to the Theory of Quantized Fields (Interscience Publishers, New York, 1959), pp. 202, 203.] Its meaning is explained by Eq. (5).

[4]The concepts "remote past" and "remote future" are assumed to have an absolute invariant significance. For space-times in which this is not the case, alterations will be required in our results.

[5]Here the covariant derivatives are taken with respect to the original metric.

[6]$G^{\mu\nu}$ has the coordinate transformation law,

$$\delta G^{\mu\nu} = (G^{\mu\nu}\xi^{\sigma})_{,\sigma} - \xi^{\mu}{}_{,\sigma} G^{\sigma\nu} - \xi^{\nu}{}_{,\sigma} G^{\mu\sigma}.$$

[7]B. S. DeWitt and R. W. Brehme, Ann. Phys. (to be published).

[8]J. Hadamard, Lecture's on Cauchy's Problem in Linear Partial Differential Equations (Yale University Press, New Haven, 1923).

[9]The reciprocity relations for the laws of propagation are proved in reference 8. The corresponding identities for the explicit propagation functions are proved in reference 7.

ERRATA

SEARCH FOR THE REACTION $\mu^+ + e^- \rightarrow \gamma + \gamma$. C. M. York, C. O. Kim, and W. Kernan [Phys. Rev. Letters 3, 288 (1959)].

Equation (4) should read:

$$R_{\mu} = (\pi/2)\alpha\alpha_1 \lambda_e^{\,2} \rho c/(\pi a_0)^3.$$

This changes the numerical result for α_1 in Eq. (5) to $\alpha_1 < 1.1\times10^{-12}$. The authors are indebted to Mr. J. Good and Professor Y. Nambu for calling their attention to this error.

PARTIAL-WAVE DISPERSION RELATIONS FOR MESON-NUCLEON SCATTERING. Reinhard Oehme [Phys. Rev. Letters 4, 246 (1960)].

The second line below Eq. (3) should read: "due to $s \geq (m+\mu)^2$; (πN) from 0 to $+\infty$ in II and ..."

In Eq. (8), N and M should be interchanged. The second line of reference 5 should read: "become complex for positive or negative values of β. The ..."

FERROMAGNETIC RELAXATION MECHANISM for M_z IN YTTRIUM IRON GARNET. M. Sparks and C. Kittel [Phys. Rev. Letters 4, 232 (1960)].

The line below Eq. (3) should read:

where $F(\theta) = 1 + (17/2)\sin^2\theta - (35/4)\sin^4\theta$.

In paragraph 6, $T \gg 0.3$°K should read $T \gg 1.3$°K. We would like to thank Dr. R. C. Le Craw for calling our attention to this error. In the seventh paragraph, the corrected values of $1/T_1$ are $0.98\times10^5\, T_{sp}$ sec^{-1} and $0.8[8]\times10^5\, T_{sp}$ sec^{-1}, respectively.

ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

QUANTUM MECHANICAL TRANSPORT THEORY. I. INCOHERENT PROCESSES. Kenneth M. Watson, Department of Physics, University of California, Berkeley, California (Received January 14, 1958).

The transport of particles through a scattering medium is studied. A generalization of a technique due to Placzek and Wick is used to handle sums over states of excitation of the medium. The collision processes which occur are classified as "inelastic," "elastic," and "quasi-elastic" and correspond to different orderings of the Placzek-Wick series. The inelastic scatterings are described by an essentially classical transport equation and the elastic scatterings by assigning a refractive index to the medium. The "quasi-elastic" scattering involves the excitation of low-lying states of the scattering system. The coherent interference of waves scattered from near-by scatterers is important in this case and depends upon the structure of the medium. In this paper the general theory is developed in terms of a systematic sequence of approximations, of which the first gives just the classical form of transport theory. The correction terms then appear as quantum-mechanical corrections to the classical transport problem.

THERMAL CONDUCTIVITY OF ISOTOPIC MIXTURES OF SOLID HELIUM. Edward J. Walker and Henry A. Fairbank, Laboratory of Marine Physics, Yale University, New Haven, Connecticut (Received December 2, 1959).

The thermal conductivity of solid He^4 containing zero, 0.56, 1.38, and 2.8% He^3 has been measured as a function of temperature from 1.1 to 2.1°K. For pure He^4, the results are in satisfactory agreement with those of Webb, Wilkinson, and Wilks. The addition of 1.38% He^3, at a sample density $\rho = 0.208$ g cm^{-3}, caused a decrease in thermal conductivity by a factor of 5 at 1.1°K. The additional thermal resistance caused by adding He^3 was substantially independent of temperature, rather than proportional to temperature, as would be expected from isotropically distributed point scatterers. From this fact and from the magnitude of the isotope resistance it is inferred, using Klemens theory, that the He^3 is arranged on lines in the solid. A possible mechanism is that the He^3 has been attracted to, and deposited along, dislocations in the He^4 matrix.

VAPOR PRESSURES OF He^3 - He^4 MIXTURES. S. G. Sydoriak and T. R. Roberts, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received December 7, 1959).

Vapor pressures, P_X, of He^3 - He^4 mixtures ranging in liquid mole fraction, X, from 0.1 to 0.9 have been measured between 0.6° and 2.4°K versus the vapor pressure, P_3^0, of liquid He^3. Except for sharp breaks in the vicinity of the lambda and stratification temperatures of some of the mixtures, P_X/P_3^0 is found to vary slowly and smoothly with X and with temperature. Contrary to much of the work of other authors, there are no breaks at the He^4 lambda temperature.

A comprehensive smoothed table of P_X/P_3^0 is derived. Using this as a reference it is possible, for the first time, to intercompare all of the previously existing data on P_X. The data of some authors are in excellent agreement with our table but other data are in serious quantitative disagreement near 1.2°K and much previous data are in qualitative disagreement at the He^4 lambda temperature.

STABILITY OF UNIFORM PLASMAS WITH RESPECT TO LONGITUDINAL OSCILLATIONS. Peter D. Noerdlinger, Normal Bridge Laboratory, California Institute of Technology, Pasadena, California (Received December 2, 1959).

It is possible to relate the dispersion formula for longitudinal oscillations in an infinite, uniform, collision-free plasma with no magnetic field to the complex potential of a line charge distribution on the real axis of the phase velocity ($u = \omega/k$) plane. If the initial velocity distribu-

tion averaged over directions orthogonal to the direction of propagation is $f_0(v)$, the plasma is stable if and only if

$$U(u) = P\int_{-\infty}^{\infty} \frac{f_0'(v)\,dv}{v-u}$$

is negative at the minima of $f_0(v)$ on the real axis, with unimportant exceptions. In particular it is shown that single-peaked distributions are stable, while those with very sharp (e.g., non-differentiable) minima or with a zero of f_0 between two peaks are not. The charge analogy yields information on the wavelengths for which oscillations can grow and on rates of growth. Examples are given, including the case of two identical interpenetrating hot plasmas. A limited generalization to transverse oscillations is given.

NEGATIVE CURRENT-VOLTAGE CHARACTERISTICS IN HYDROGEN AT HIGH PRESSURE USING PLANE PARALLEL ELECTRODES. D. J. De Bitetto* and L. H. Fisher, Department of Physics, New York University, University Heights, New York, New York, and A. L. Ward, Diamond Ordnance Fuze Laboratories, Washington, D. C. (Received December 17, 1959).

In conjunction with measurements of current-voltage characteristics in hydrogen, a few characteristics have been obtained which include a region with negative slope. The latter characteristics were obtained with plane parallel electrodes at an electrode separation of 2 cm at a pressure of 400 mm Hg and with three values of externally initiated cathode current. The initial currents ranged from about 10^{-11} to 10^{-9} amp, and the amplified currents reached values as high as 10^{-4} amp. The characteristics corresponding to the larger initial currents become negative at large currents ($\sim 10^{-5}$ amp). The voltage at which a characteristic becomes negative, i.e., the maximum attainable voltage across the electrodes, decreases slightly with increasing initial current. The circuit included a series resistor of 20 megohms.

These characteristics can be explained quantitatively on the basis of the first and second Townsend coefficients (previously measured with the same apparatus) acting in conjunction with space charge, if a not unreasonable discharge area is assumed. These calculations were carried out on an IBM 704 computer.

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RANDOM-WALK INTERPRETATION AND GENERALIZATION OF LINEAR BOLTZMANN EQUATIONS, PARTICULARLY FOR NEUTRON TRANSPORT. E. Guth and E. Inönü, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received December 21, 1959).

The connection between linear recurrence relations which define generalized random walks and the related linear Boltzmann equations is clarified. The probability distributions $f_n(s)$ for "the state s" reached by a "random walker" after n steps satisfies the recurrence relation $f_{n+1}(t) = \int f_n(s)P(s,t)ds$, where the non-negative $P(s,t)$ is the probability for a transition from s to t. The Boltzmann distribution is given by $f(s) = \sum_{n=0}^{\infty} f_n(s)$. In general, $f_n(s)$ contains more information than $f(s)$. Moreover, $f_n(s)$ is the nth term in the iteration series solution of the Boltzmann equation and therefore can also be obtained from the solution of an associated Boltzmann equation which contains an additional parameter. As an example, the well-known integral Boltzmann equation for neutron transport in a nonmultiplying infinite medium is derived from a $P(s,t)$ which involves a transition in a seven-dimensional phase-time space. Brownian motion and Rayleigh's problem (related to neutron thermalization) may be treated similarly.

ZEEMAN EFFECT OF IMPURITY LEVELS IN SILICON. Solomon Zwerdling, Kenneth J. Button, and Benjamin Lax, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts (Received December 28, 1959).

Completely resolved Zeeman spectra for the bismuth donor in silicon including optical transitions from the $1s$ donor ground state to the excited states $2p_0$, $2p_\pm$, $3p_0$, $4p_0$, $3p_\pm$, $5p_0$, $4p_\pm$, and $5p_\pm$ are presented. The transitions were observed at liquid helium temperature, using linearly polarized radiation alternately parallel and perpendicular to the magnetic field, and field intensities up to 38.9 kilogauss oriented along each of the three principal crystallographic axes. Both linear splitting of the $p_\pm$ states and

a quadratic dependence on field were observed. The use of impurity Zeeman spectra is demonstrated for evaluating effective mass parameters, determining the nature of energy bands and finding and identifying impurity excited states. The transverse effective mass for the electron in silicon was found to be $(0.186 \pm 0.006)m_0$ in agreement with recent cyclotron resonance results. From Zeeman splitting, electron effective masses up to $0.5m_0$ can be measured to within $\pm\frac{1}{2}\%$ at infrared frequencies in a field of 40 kilogauss. The behavior in a magnetic field of the first two donor excited states could be explained by treating the magnetic terms of the Hamiltonian as a perturbation to first order. Interactions among the higher closely-spaced Zeeman levels were observed above 20 kilogauss and were evaluated with a second-order treatment. The Zeeman structure for the aluminum acceptor reflected the complexity of the valence bands and the acceptor ground state and was in qualitative agreement with the theoretical results of Kohn and Schechter. Transitions were observed to eight excited states converging to the series limit. Evidence is given for the degeneracy of each state.

CROSS RELAXATION STUDIES IN DIAMOND. P. P. Sorokin, G. J. Lasher, and I. L. Gelles, Research Laboratory, International Business Machines Corporation, Poughkeepsie, New York (Received December 18, 1959).

A microwave double resonance experiment performed on the paramagnetic nitrogen centers in diamond shows that in this system cross relaxation occurs via a four-spin-flip mechanism which exactly conserves Zeeman energy. In this process, which was first postulated by Bloembergen and co-workers in their paper on cross relaxation, two spins of the center line make a downward transition while a spin belonging to each satellite makes an upward transition. Simple rate considerations for this process indicate that if a saturating microwave field is suddenly applied to one of the three lines of the nitrogen spectrum, a weak probing microwave signal at either of the two lines should register a definite change in absorption in a time T_{21}. Specifically, if T_{21} is much less than other relaxation times of the system, then setting the pump upon the center line should force the absorption at either satellite to drop to zero. Setting the pump at the position of one of the satellites, on the other hand, should reduce the center line absorption to 3/5 its thermal equilibrium value but should increase the absorption measured at the other satellite by the factor 6/5. This behavior was precisely observed at 1.6°K, using a bi-modal cavity.

By resolving the rate at which a satellite decays to zero when the pump is set on the center line, T_{21} is measured for all five satellites in the three principal orientations: $H_0 \| [100]$, $H_0 \| [110]$, and $H_0 \| [111]$. The measured anisotropy is discussed.

It is shown that the four-spin-flip transition may be used in special cases to establish cw maser operation by inverting the population of one of the satellite lines. Steady-state inversion of one of the nitrogen satellites is incidentally observed in a number of diamonds.

INFLUENCE OF COLD WORK ON THE RESISTIVITY OF DILUTE COPPER ALLOYS. P. G. Klemens,* R. J. Tainsh, and G. K. White, Division of Physics, Commonwealth Scientific and Industrial Research Organization, Sydney, Australia (Received November 30, 1959).

Measurements are reported of the residual electrical resistivity of a copper + 1 atomic percent antimony alloy which had been cast and rolled. A large increase was observed on annealing, as noted by Kropschot, Garber, and Blatt. However, metallographic examination and the smallness of the change observed on subsequent rerolling suggest that their proposed mechanism of "migration" during cold work plays a much smaller part than lack of homogeneity in the cast alloy.

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TRANSVERSE COLLECTIVE EXCITATIONS IN SUPERCONDUCTORS AND ELECTROMAGNETIC ABSORPTION. T. Tsuneto, Department of Physics, University of Illinois, Urbana, Illinois (Received December 24, 1959).

With use of the generalized random phase approximation an attempt is made to estimate the absorption of photons with energy less than the energy gap due to transverse collective excitations. The ratio of the surface resistance

due to transverse collective excitations to that of normal metals in the extreme anomalous limit, calculated within the weak-coupling theory, turns out to be too small to explain the observed data for superconducting lead and mercury. The interpretation of the collective excitations as bound pair states is briefly discussed.

MAGNETIC SCATTERING OF NEUTRONS BY EXCHANGE-COUPLED LATTICES. A. W. Sáenz, U. S. Naval Research Laboratory, Washington, D. C. (Received September 21, 1959).

The magnetic scattering of slow neutrons of arbitrary initial polarization by an extensive class of magnetically-coupled lattices is treated by a time-dependent operator approach. This magnetic scattering is carefully divided into purely magnetic and magnetovibrational scattering, the types thereof involving, respectively, only zero-phonon processes and solely nonzero-phonon processes, and general formulas for these two types of scattering are obtained. These formulas are applied in temperature regions which are sufficiently large (I) or sufficiently small (II) compared with the temperature above which paramagnetism obtains. In region I, for the purely magnetic scattering and under certain invariance requirements on the above magnetic coupling, we analyze the energy spectrum of outgoing initially unpolarized neutrons of sufficiently high incident energy by a moment method. We thus obtain general formulas for the energy-integrated effective differential cross section defined in this paper and for the moments of energy transfer defined therein. These formulas involve certain spin averages, explicit equations being given for a wide range of these averages for exchange-coupled lattices. These results are illustrated numerically and compared with experiment for the case of polycrystalline MnF_2. In region II, we discuss certain broad features of the purely magnetic one-magnon scattering of arbitrarily polarized neutrons by exchange-coupled lattices of the class alluded to previously and by more complex ones, studying this scattering in detail for ferromagnets and certain antiferromagnets. A new spin-wave effect is pointed out for polarized neutrons incident on ferromagnets. Brief treatments of the magnetovibrational scattering in regions I and II are also given.

THEORY OF NUCLEAR QUADRUPOLE INTERACTION IN BERYLLIUM METAL. M. Pomerantz and T. P. Das,* Department of Physics, University of California, Berkeley, California (Received December 29, 1959).

The theory of the origin of the field gradient at nuclei in metals has been analyzed. The contributions of the ion cores and conduction electrons have been separately considered. In the case of beryllium metal, using orthogonalized plane wave functions, the conduction electrons are shown to enhance, by about eight percent, the field gradient due to the ion cores. Combining the results of our calculations with Knight's experimental value of 48 kc/sec for the Be^9 coupling constant e^2qQ/h, a value of $Q = 0.029 \times 10^{-24}$ cm^2 is obtained. The dependence of the potential for the conduction electrons on the model chosen is analyzed in some detail. The various uncertainties in our field-gradient calculation and the theoretical value of the Knight shift in beryllium metal are discussed.

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MEASUREMENT OF LIFETIME IN Ge FROM NOISE. Susumu Okazaki and Hiroshi Oki, Department of Physics, Faculty of Science, Okayama University, Okayama, Japan (Received December 14, 1959).

The lifetime of the minority carrier can be obtained from a noise measurement whose method consists of liberating hole-electron pairs by light. A convenient experimental arrangement is suggested and a typical plot of the experimental results is shown. The curves of the results are compared with the calculated curves.

THE ELECTRONIC STRUCTURE OF TIN INVESTIGATED BY ULTRASONIC ATTENUATION. Tore Olsen,* Department of Physics, Brown University, Providence, Rhode Island, and Fysisk Institutt, Universitetet i Oslo, Blindern, Norway (Received December 1, 1959).

The magnetic field dependence of the ultrasonic attenuation has been measured in very pure tin single crystals. Oscillations were found that

can be explained as a result of resonant conditions between the electron orbit diameter and the periodic field set up by the sound wave. These oscillations yield information about the Fermi momentum, and the general features of a possible electron distribution in tin are suggested.

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SATURATION AND RECOVERY OF A PORTION OF THE ELECTRON SPIN RESONANCE OF F CENTERS IN KCl AT 4°K. Gordon A. Noble, Research Department, Zenith Radio Corporation, Chicago, Illinois (Received December 14, 1959).

The inhomogeneously broadened magnetic resonance of F centers in additively colored potassium chloride was photographed using a field modulation which swept rapidly across the resonance as the absorption was displayed on the oscilloscope. The time between sweeps was long and adjustable. A concentration dependance of the spin lattice relaxation time was observed at 4.2°K. In crystals with a concentration near 1×10^{17} cm^{-3}, a portion of the resonance could be saturated without affecting the rest of the broad spectrum. The recovery time of this "hole burned in the resonance" is about a quarter of a minute. The results are compared with present theories on spin diffusion.

EFFECTS OF HYDROSTATIC PRESSURE ON THE PIEZORESISTANCE OF SEMICONDUCTORS: i-InSb, p-Ge, p-InSb, AND n-GaSb. R. W. Keyes and M. Pollak,* Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received December 10, 1959).

A method for measuring the piezoresistance of a sample under high hydrostatic pressure by comparison with the piezoresistance of intrinsic InSb is described. The method is tested by a measurement of the piezoresistance of p-type germanium and p-type InSb up to 13 000 kg/cm^2. The piezoresistance of these materials is found to be independent of pressure, in good agreement with predictions based on other experiments. Measurement of the piezoresistance of n-GaSb as a function of pressure up to 12 000 kg/cm^2 confirms the correctness of the model of the conduction band which has been proposed by Sagar. Values for certain parameters of the conduction band of GaSb are deduced.

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SURFACE IMPEDANCE OF SUPERCONDUCTORS. Piotr B. Miller, Department of Physics, University of Illinois, Urbana, Illinois (Received December 21, 1959).

A detailed calculation of the surface impedance of superconductors is given based on the general theory of the anomalous skin effect in normal and superconducting metals given by Mattis and Bardeen. It is found that there are large corrections to the extreme anomalous limit value of the superconducting to normal surface resistance ratio; corrections to the surface reactance ratio are much smaller. The theory is compared with recent experiments on the surface impedance of aluminum and of tin. It is found that the theory gives satisfactory agreement with experimental data on the surface impedance, both in absolute value and in its temperature and frequency dependence over a wide range of temperatures and frequencies.

HEAT CAPACITY OF SODIUM AND POTASSIUM AT TEMPERATURES BELOW 1°K. William H. Lien and Norman E. Phillips, Department of Chemistry and Lawrence Radiation Laboratory, University of California, Berkeley, California (Received December 17, 1959).

The heat capacities of sodium and potassium have been measured from about 0.15°K to just above 1°K. No anomaly was observed in either case: the heat capacity could be represented by the sum of a term linear in temperature and a cubic term. Values of the Debye temperatures and effective masses of the conduction electrons are given.

TRANSPORT NUMBER IN SOLID CESIUM BROMIDE. Neal Laurance, Department of Physics, University of Illinois, Urbana, Illinois (Received December 9, 1959).

The transport number of cesium bromide single crystals has been measured over the temperature range 350° to 450°C by the method of

Tubandt. No temperature dependence is observed, and the average value of the cation transport number is 0.49 ± 0.05. A preferential growth phenomenon is described, and its possible interference with the measurement is discussed. It is estimated that the true cation transport number may be as low as 0.3 because of this phenomenon.

RECTIFICATION WITHOUT INJECTION AT METAL-TO-SEMICONDUCTOR CONTACTS. N. J. Harrick, Philips Laboratories, Irvington-on-Hudson, New York (Received December 7, 1959).

A recent publication shows that extraction in the semiconductor bulk may occur for either direction of current flow through the same metal-to-semiconductor contact when an insulating layer separates the metal and the semiconductor and the field-effect determines the surface barrier under the metal. It is shown here that strong rectification, whose direction depends only on the bulk type, may occur for such contacts to extrinsic, but not intrinsic, semiconductors. Thus, rectification, without injection, may occur at the metal-to-semiconductor contact for the two-carrier system.

DIFFUSION OF CADMIUM AND ZINC IN GALLIUM ARSENIDE. Bernard Goldstein, Radio Corporation of America Laboratories, Princeton, New Jersey (Received December 4, 1959).

The diffusion of Cd and Zn in GaAs has been studied by using radioactive isotopes of these elements as tracers. The diffusion of Cd follows the correct solution to the diffusion equation and its temperature dependence is of the customary form, $D = D_0 \exp(-E/kT)$, where E, the activation energy, is 2.43 ev and D_0 is 0.05 cm^2/sec. The diffusion of Zn from the vapor cannot, however, be described in terms of a single diffusion constant. The penetration curves decrease much more sharply than they theoretically should. Hall measurements indicate that all the Zn is substitutional and that it forms an impurity conduction band merging with the valence band. When Zn diffuses from a thin electroplated layer of radio-zinc, then the penetration profiles do correspond to the proper solution to the diffusion equation. The diffusion constants so determined have the usual temperature dependence given by $D = D_0 \exp(-E/kT)$, when D_0 is 15 cm^2/sec and E is the same as that found for Cd. From the work reported here and that of others, it is suggested that the diffusion of Cd and Zn in GaAs proceeds via vacancy migration within the gallium sublattice.

SURFACE TRANSPORT IN SEMICONDUCTORS. R. F. Greene, United States Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland, D. R. Frankl, Central Research Laboratories, Sylvania Electric Products, Incorporated, Bayside, New York, and Jay Zemel, United States Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland (Received November 13, 1959).

A transport theory is given for electrons and holes in space-charge layers at semiconductor surfaces. For diffuse surface scattering, the effective surface mobilities may differ significantly from the bulk mobility for any strength of space-charge layer. Agreement with Schrieffer's formulas is found only for strong space-charge layers, and the discrepancy is explained. The results are extended to cover an arbitrary degree of diffuseness of surface scattering and to cover samples of small thickness.

FAR INFRARED TRANSMISSION THROUGH SUPERCONDUCTING FILMS. D. M. Ginsberg and M. Tinkham, Department of Physics, University of California, Berkeley, California (Received December 30, 1959).

The far infrared transmission through films of superconducting and normal lead, tin, indium, and mercury has been measured in the wavelength region between 0.1 and 1.1 mm. The transmission data have been analyzed to find the ratio of the complex conductivity in the superconducting state to that in the normal state, as a function of frequency. The width of the energy gap at 0°K may be estimated from the frequency of the extrapolated cutoff of the real part, $\sigma_1(\omega)$, of the superconducting conductivity. The values so obtained are 4.0 ± 0.5, 3.3 ± 0.2, and $3.9 \pm 0.3\ kT_c$ for lead, tin, and indium, respectively. These values are in good agreement with those obtained in other experiments on bulk samples. The frequency dependence of $\sigma_1(\omega)/\sigma_N$

is in qualitative agreement with the results of a calculation by Mattis and Bardeen based on the theory of Bardeen, Cooper, and Schrieffer, except for an unexpected hump in $\sigma_1(\omega)$ for lead and (tentatively) mercury at low frequencies. This hump may be due to the production of collective excitations or an anisotropy in the energy gap. It has also been found that a magnetic field as high as 8000 gauss applied in the plane of a lead film about 12 A thick has only a very small effect on the electromagnetic properties of the film. This is not surprising, in view of the results of the microwave experiments of Pippard and of Spiewak.

ATOMIC HEAT OF SODIUM METAL FROM 0.4 TO 2°K. R. E. Gaumer and O. V. Heer, Department of Physics, Ohio State University, Columbus, Ohio (Received November 9, 1959).

The atomic heat of sodium metal was measured from 0.4 to 2°K using the magnetic refrigerator calorimeter. The experimental data are given by $C=1.32T+0.485T^3$ millijoule/mole-°K. No anomaly is observed in this temperature range. The experimental value of the electronic specific heat and the Pauli spin paramagnetism measured by Schumacher and Slichter are used for comparison with the electronic band theory of sodium and those modifications introduced by electron-electron and lattice-electron interactions.

TWO-PHONON INDIRECT TRANSITIONS AND LATTICE SCATTERING IN Si. W. P. Dumke, International Business Machines Research Laboratory, Poughkeepsie, New York (Received December 30, 1959).

The probability of indirect transitions with the emission of two phonons has been calculated. Several of the intensity maxima in the intrinsic low-temperature emission spectrum of Si are explainable in terms of these transitions. Scattering matrix elements obtained from an analysis of the observed emission spectrum indicate that intervalley scattering is the dominant scattering mechanism for electrons in Si, with 0.023-ev and 0.046-ev longitudinal acoustic mode phonons umklapp scattering electrons between valleys on the same and on different crystal axes, respectively. The valleys are approximately 82% of the way from the center to the edge of the Brillouin zone. Optical mode scattering in the valence band is largely responsible for the anomalous temperature dependence of the intrinsic hole mobility.

VARLEY MECHANISM FOR DEFECT FORMATION IN ALKALI HALIDES. D. L. Dexter, University of Rochester, Rochester, New York (Received December 2, 1959).

The Varley mechanism is examined, according to which Frenkel defects are produced in the halogen sublattice of alkali halides subsequent to multiple ionization of the halide ions. Arguments are presented to show that the lifetime of a positive halogen ion against recapture of electrons is orders of magnitude smaller than the ejection time of the halogen, and thus that the Varley mechanism is inoperative. The arguments may not be applicable for inner shells alone, but experimental evidence is adduced to eliminate this case.

SOME OPTICAL PROPERTIES OF CADMIUM TELLURIDE. P. W. Davis and T. S. Shilliday, Battelle Memorial Institute, Columbus, Ohio (Received December 28, 1959).

Some optical properties of CdTe samples having a carrier concentration of the order of 10^{15} cm^{-3} were measured. The index of refraction in the infrared was determined to be 2.61. Values of the absorption coefficient α as a function of frequency were obtained at various temperatures. By plotting $\alpha^{1/2}$ and α^2 against frequency, it was found that at room temperature the minimum energy for direct transitions is 1.50 ev; that for indirect transitions is 1.44 ev. The temperature dependencies between 77°K and 373°K are given by $(1.66-5.6\times10^{-4}T)$ ev for direct transitions and $(1.56-4.1\times10^{-4}T)$ ev for indirect transitions.

LOW-TEMPERATURE PROPERTIES OF H, V, AND F CENTERS IN KCl AND KBr. John Cape and Gilbert Jacobs,* Department of Physics, University of Illinois, Urbana, Illinois (Received December 9, 1959).

Potassium chloride and potassium bromide crystals were exposed to x rays at 10°K. The optical absorption produced by this irradiation

and the changes in optical absorption produced by subsequent annealing at higher temperatures were measured. The temperatures at which changes in optical absorption occurred were correlated with the temperatures at which free electrical charge appeared, and thermoluminescence was observed. The absorption band at 345 mμ, in KCl, which has been named the *H* band, is shown to result from the superposition of two or more bands, one of which is the absorption band due to self-trapped holes. In KCl, the self-trapped hole band bleaches thermally at 43°K with a release of free electrical charge. *H* centers disappear at 56°K with a release of free charge. The optical absorption band of the *H* center is shown to have its maximum at 335 mμ. In KBr, the thermal release of free charge at 30°K is attributed to the disappearance of *H* centers. No charge burst was observed in KBr, which may be attributed to the destruction of self-trapped holes.

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FREQUENCY FACTOR AND ENERGY DISTRIBUTION OF SHALLOW TRAPS IN CADMIUM SULFIDE. James J. Brophy and Robert J. Robinson, Physics Division, Armour Research Foundation, Chicago, Illinois (Received December 10, 1959).

Current noise and photoconductivity measurements taken under uniform 5200A illumination on CdS single crystals are used to derive the energy distribution and frequency factor of shallow traps in the range 0.3 to 0.6 electron volt below the conduction band for samples of different CuCl impurity content. Trap densities varying from 10^{12} to 10^{17} cm^{-3} ev^{-1} and total trap concentrations of 10^{16} cm^{-3} with discrete levels at 0.36, 0.43, and 0.60 ev below the conduction band are observed. In a moderately doped, good photosensitive crystal, the traps also have a continuous distribution in energy and all have the same frequency factor, 10^{11} sec^{-1}, which suggests that the traps are structurally similar. The results imply that a photoelectron may experience several thousand retrapping transitions on the average before recombining. It is possible to account semiquantitatively for the $1/f$ noise spectrum observed in some crystals at high frequencies in terms of the near-exponential trap distributions and constant frequency factor derived from low-frequency noise measurements.

CUBIC FIELD SPLITTING OF *D* LEVELS IN METALS. Joseph Callaway, Department of Physics, University of Miami, Coral Gables, Florida, and D. M. Edwards, Department of Mathematics, Queen Mary College, University of London, London, England (Received December 8, 1959).

The splitting of the fivefold degeneracy of free-atom *d* electron states by nonspherical components of a crystalline field is calculated. The crystal potential employed is that of a lattice of positive point charges screened by a uniform distribution of electrons. The calculation is done to first order in the cubic field, using hydrogenic electron wave functions. The triply degenerate *d* state is lowered with respect to the doubly degenerate one in both body-centered and face-centered cubic lattices. Numerical results are given for both lattices. Finally, analytic atomic wave functions are used to estimate the splitting in iron and copper at the observed lattice spacing. The crystal field splitting of these levels is found to be much smaller than the overlap splitting as obtained in previous calculations for both materials.

NEUTRON DIFFRACTION STUDIES OF ANTIFERROMAGNETISM IN CrF_2 AND $CrCl_2$. J. W. Cable, M. K. Wilkinson, and E. O. Wollan, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received December 11, 1959).

Neutron diffraction observations have been made on powder samples of CrF_2 and $CrCl_2$ from 298° to 4.2°K. These materials, which have crystal structures similar to rutile but distorted from tetragonality, become antiferromagnetic at low temperatures with different magnetic structures. For CrF_2 the magnetic unit cell is identical with the chemical cell and the moments at the corner sites are directed oppositely to those at the center of the cell. The magnetic unit cell of $CrCl_2$ requires doubling of the *b* and *c* axes of the orthorhombic chemical cell and the structure consists of ferromagnetic (011) planes with adjacent planes antiparallel. In neither case were the intensities compatible with a magnetic axis directed along a simple crystallographic direction. Unique magnetic axes could not be definitely established but the data suggest that they lie parallel to the longest Cr-F and Cr-Cl bonds. Néel temperatures of

53° and 20°K were observed for CrF_2 and $CrCl_2$, respectively.

GROUND-STATE ENERGY AND GREEN'S FUNCTION FOR REDUCED HAMILTONIAN FOR SUPERCONDUCTIVITY. J. Bardeen and G. Rickayzen,* Department of Physics, University of Illinois, Urbana, Illinois (Received December 14, 1959).

In their theory of superconductivity, Bardeen, Cooper, and Schrieffer made use of a reduced Hamiltonian which included only scattering of pairs of particles of opposite momentum and spin. It is shown that the solution they obtained by a variational method is correct to $O(1/n)$ for a large system. The single-particle Green's function is derived and used to calculate the interaction energy.

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IRON SERIES HARTREE-FOCK CALCULATIONS. R. E. Watson, Solid State and Molecular Theory Group, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received November 10, 1959).

Seventy-six Hartree-Fock calculations have been completed for the iron series (Sc to Cu) atoms and ions. All calculations are for the $(3d)^n$ type of configuration (i.e., no 4s electrons are present). The results are discussed, but due to lack of space, are not presented in full here. Comparisons are made with the experimental ionization and multiplet spectra. Agreement is poor due to limitations in the Hartree-Fock formalism. The results are used in an effort to gain information concerning correlation energies.

DISPERSION RELATIONS IN ATOMIC SCATTERING PROBLEMS. E. Gerjuoy and N. A. Krall, John Jay Hopkins Laboratory for Pure and Applied Science, General Atomic, San Diego, California (Received December 17, 1959).

Dispersion relations appropriate to the scattering of electrons by hydrogen atoms are deduced, and applied to actual measurements in the 0 - 10 ev energy range. Two such experiments exist, yielding quite different results. Dispersion relations indicate that only certain angular distributions at low energy, and certain scattering amplitudes at high energy are consistent with these low-energy total cross-section measurements; this suggests experiments which could be used as checks on the accuracy of the existing measurements.

BETA-GAMMA ANGULAR CORRELATION MEASUREMENTS ON Au^{198}. I. DIRECTIONAL AND CIRCULAR POLARIZATION CORRELATION. Rolf M. Steffen, Department of Physics, Purdue University, Lafayette, Indiana (Received September 2, 1959).

The beta-gamma directional correlation of Au^{198} has been measured as a function of the energy of the beta particles. The anisotropy factor $A_2(W)$ in the beta-gamma directional correlation $W_{\beta\gamma}(\theta, W) = 1 + A_2(W)P_2(\cos\theta)$ is proportional to p^2/W and its value near the maximum beta energy is $A_2(2.8) = 0.029 \pm 0.001$. The beta directional gamma circular polarization correlation of Au^{198} was measured using the backscattering of the circularly polarized gamma radiation from a radially magnetized iron disk. The correlation is of the form: $W_{\beta\gamma\mathrm{pol}}(\theta, W) = 1 + (0.45 \pm 0.07)(p/W)P_1(\cos\theta)$. The shape correction factor of the first-forbidden beta spectrum is independent of the beta energy above $W = 1.6$ within 5%.

The analysis of the data shows that the ξ-approximation for first-forbidden nonunique beta transitions represents the Au^{198} results in a very satisfactory manner.

BETA-GAMMA ANGULAR CORRELATION MEASUREMENTS ON Au^{198}. II. TRANSVERSE POLARIZATION OF THE BETA PARTICLES. P. C. Simms* and R. M. Steffen, Department of Physics, Purdue University, Lafayette, Indiana (Received October 15, 1959).

As a consequence of the nonconservation of parity in beta decay, beta particles emitted in first-forbidden beta transitions exhibit a small degree of polarization transverse to their momentum. The direction of the transverse polarization is defined with respect to a plane which is introduced by observing the direction of emission of the beta particles and the direction of

emission of a gamma ray following the beta transition. The degree of the transverse polarization parallel to the beta-gamma plane $P_{T\parallel}$, and the degree of polarization perpendicular to the beta-gamma plane $P_{T\perp}$ have been measured for Au^{198}. The beta polarization has been detected by means of the left-right asymmetry in a Mott scattering process for an average electron energy $W = 2.0\ mc^2$ and for an angle $\Theta = 135°$ between the beta momentum and the gamma direction. The results of the measurements, $P_{T\parallel} = +0.011 \pm 0.005$ and $P_{T\perp} = 0.03 \pm 0.008$, agree satisfactorily with the values calculated on the basis of the ξ approximation from the anisotropy of the Au^{198} beta-gamma directional correlation.

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ELASTIC SCATTERING OF 5.6-Mev NEUTRONS FROM CARBON. J. E. Braley and C. W. Cook, Convair, San Diego, California (Received October 26, 1959).

Energetic neutrons produced from the $D(d,n)He^3$ reaction have been scattered from a cylindrical carbon sample to study the angular distribution of elastically scattered 5.6-Mev neutrons. The differential elastic scattering cross sections for carbon have been obtained for angles in the range of 30 to 150 degrees. A thin plastic neutron-proton recoil detector was used in the measurements to permit discrimination against 4.4-Mev gamma rays from the carbon sample, and other gamma-ray backgrounds.

RADIOACTIVE DECAY OF Lu^{172}. R. G. Wilson and M. L. Pool, Department of Physics and Astronomy, Ohio State University, Columbus, Ohio (Received October 5, 1959; revised manuscript received January 11, 1960).

Ytterbium oxide enriched to 95.9% in the 172 mass number was irradiated with 6-Mev protons. An activity decaying by electron capture with a half-life of 6.70 ± 0.04 days was produced and its assignment to Lu^{172} confirmed by the identification of the ytterbium K x ray and by comparison with the activities produced by similar proton irradiations of the other enriched isotopes of ytterbium. The 4.0-hour positron activity previously assigned to Lu^{172} was not observed and is best attributed to an impurity. The observed activity of Lu^{172} consists of the L and K x rays of ytterbium and gamma rays with energies of 79, 91, 113, 182, 203, 270, 324, 373, 490, 527, 697, 809, 900, 912, 1093, 1402, and 1583 kev. Because no positron radiation exists in the activity of Lu^{172}, the mode of decay is solely by electron capture to Yb^{172}. Gamma-gamma coincidence measurements, energy considerations, and consideration of the relative numbers of the radiations observed in the activity of Lu^{172} have led to the assignment of energy levels at 530 (6+), 1172 (3+), 1263 (4+), 1375 (5+), 1662 (3-), (1699), and 2072 (4+) kev in Yb in addition to the previously known 78.7 (2+)- and 260.2 (4+)-kev levels. The positions of all of the observed radiations and some observed only in conversion electron measurements are shown in a proposed energy level scheme for the decay of Lu^{172}. Approximate branching ratios for the electron capture disintegrations of Lu^{172} are also shown in the level scheme. Few if any electron capture transitions of Lu^{172} occur to the ground and first excited states of Yb^{172}. Of the two predicted spins for the ground state of Lu^{172}, 4- is more consistent with the proposed energy level scheme.

MULTIPLE SCATTERING OF POLARIZED ELECTRONS. M. K. Sundaresan, Department of Physics, Panjab University, Chandigarh, India (Received December 29, 1960).

The correct theory of multiple scattering is used to evaluate the numerical magnitudes of the depolarization produced due to multiple scattering in gold foil of thickness 1 mg/cm^2 for polarized electrons of velocities: $v/c = 0.6, 0.7, 0.8, 0.9$. The depolarization effect is found to be extremely small. The correction due to multiple scattering to the electrostatic rotation of spins is also computed.

DECAY OF THE $i_{13/2}$ STATE IN Pb^{205}. R. Stockendal, Nobel Institute of Physics, Stockholm, Sweden (Received December 21, 1959).

The $L_{\rm I}$, $L_{\rm II}$, $L_{\rm III}$, $M_{\rm I}$, $M_{\rm II}$, $M_{\rm III}$, $M_{\rm IV}$, $N_{\rm I}$, $N_{\rm III}$ and $O_{\rm I}$ conversion lines of a 26.22 ± 0.01 kev transition in Pb^{205} have been found from studies with electromagnetically separated samples of Bi^{205}. The transition, having $M2$ character, most probably takes place from a 4-msec $i_{13/2}$ state of 1014.0 kev to the 987.8-kev 9/2- state, thus

causing the latter state to appear metastable, as has earlier been reported. This suggestion is also supported by strong evidence for the existence of a 310.5-kev $E3$ transition between the isomeric state and the 703.3-kev state. The energy 1014.0 kev of the $i_{13/2}$ state coincides with a transition energy in Pb^{205} earlier reported, and is found to fit well into the energy systematics of the $i_{13/2}-f_{5/2}$ $M4$ transitions in the odd lead isotopes.

ISOMERIC TRANSITION IN Pb^{205}. David E. Alburger, Brookhaven National Laboratory, Upton, New York (Received December 30, 1959).

A (26 ± 1)-kev transition in Pb^{205} occurring in the electron capture decay of Bi^{205} has been identified from its L, M, and N internal conversion electrons measured in an intermediate-image beta-ray spectrometer. By using NaI scintillation detectors behind the source in the spectrometer, the M and N lines of the 26-kev transition are found to be not in coincidence with electron-capture K x rays but they are in coincidence with a principal gamma ray of 1 Mev and weak components of ~0.7 and ~0.3 Mev. Since the Bi^{205} gamma rays of 987.8 and 284.2 kev and a fraction of the 703.3-kev gamma rays have been shown by Vegors and Heath to be delayed with respect to electron capture K x rays with a half-life of 4.8 msec, the present coincidence results indicate that the delayed radiation is associated with the 26-kev transition originating from an isomeric state in Pb^{205} at 1013.8 kev. The 26-kev transition is probably a quadrupole and a possible assignment is $M2$ if the spin-parity assignment of the 987.8-kev level is 9/2- and if the 1013.8-kev level is the 13/2+ state predicted at 1.1 Mev by Pryce. The possibility that all or part of the known 1014.2-kev gamma radiation constitutes the "missing" $M4$ transition in Pb^{205} is discussed.

DECAY OF Sm^{155}. R. E. Sund, R. G. Arns, and M. L. Wiedenbeck, Harrison M. Randall Laboratory of Physics, The University of Michigan, Ann Arbor, Michigan (Received November 25, 1959).

Sm^{155} was found to decay to Eu^{155} with a half-life of 21.9 ± 0.2 minutes. Internal conversion electrons corresponding to the gamma rays in Eu^{155} were observed in a magnetic spectrometer. The gamma-ray spectrum was studied with a well crystal. The spectra of gamma rays coincident with the x ray, 104-kev, 142-kev, and 246-kev gamma rays were observed. A number of new, weak transitions are proposed. Directional correlation measurements were made on the 142-kev–104-kev cascade. Possible spin assignments are discussed.

RANGE-ENERGY RELATIONS FOR PROTONS IN VARIOUS SUBSTANCES. R. M. Sternheimer, Brookhaven National Laboratory, Upton, New York (Received December 30, 1959).

An expression is obtained for the range-energy relation $R(T_p)$ for protons (T_p = proton kinetic energy) as a function of the mean excitation potential I which enters into the Bethe-Bloch formula for the ionization loss dE/dx. The expression for $R(T_p)$ is obtained by an interpolation of the previously calculated range-energy relations for Be, Al, Cu, and Pb. The resulting expression for $R(T_p)$ can be used for any substance, provided an appropriate value of I is assumed. Values are also obtained for the quantity $q=(I/R)(dR/dI)$ which gives the fractional change of R for a small variation of the excitation potential I.

DIRECTIONAL CORRELATION OF THE GAMMA RAYS IN W^{182}. G. D. Hickman* and M. L. Wiedenbeck, Harrison M. Randall Laboratory, University of Michigan, Ann Arbor, Michigan (Received December 14, 1959).

Directional correlation measurements were performed on seven cascades involving the gamma rays in W^{182}. The angular momenta of the excited states and the character of the gamma transitions which were determined by the correlation measurements are in good agreement with the values as determined by previous investigators. There is evidence for the existence of three rotational bands with the possibility of a fourth band in W^{182}. The nature of these rotational bands is explained in terms of the Bohr-Mottelson model for spheroidal nuclei. The levels are characterized by the quantum numbers (K, I, π). After correction for x-ray interference the 1.222 Mev-0.068 Mev correlation results in a spin of 3 to the 1.290-Mev

level. The 0.068-Mev gamma ray is essentially pure dipole radiation. The results of the 1.231 Mev-0.100 Mev correlation are in good agreement with spin 3 to the 1.331-Mev level with a quadrupole content of $(2 \pm 0.5)\%$ for the 1.231-Mev radiation. The 0.152 Mev-1.222 Mev correlation is only in agreement with an assignment of spin 3 to the 1.374-Mev level with a quadrupole content of less than 0.5% for the 0.152-Mev transition. The 0.152 Mev-1.122 Mev correlation yields a quadrupole content of (3 - 11% or 94 - 99%) for the 1.122-Mev transition. The 0.222 Mev-1.231 Mev, 0.264 Mev-1.222 Mev correlations are in good agreement with the assignment of spin 4 to the 1.554-Mev level. However, spins of 3 or 5 for the 1.554-Mev level are also in agreement with the correlation data.

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PROTON-PROTON SCATTERING AT 25 Mev. T. H. Jeong, L. H. Johnston, and D. E. Young,* University of Minnesota, Minneapolis, Minnesota, and C. N. Waddell, University of Southern California, Los Angeles, California (Received December 28, 1959).

The differential cross section for proton-proton scattering has been measured for 23 center-of-mass angles from 10° to 90°, with ± 0.8% absolute probable error at angles greater than 14°. The incident proton energy was 25.63 Mev lab. The 90° cross section is 18.59 millibarns, and the interference minimum of 17.09 mb occurs at 24° c.m. A set of phase shifts which fit the data are: $^1S_0$, 49.5°; $^3P_0$, 8.2°; $^3P_1$, -4.2°; $^3P_2$, 2.0°; $^1D_2$, 0.62°.

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PROTON POLARIZATION IN p-d SCATTERING. S. M. Shafroth,* R. A. Chalmers, and E. N. Strait, Northwestern University, Evanston, Illinois, and R. E. Segel, Aeronautical Research Laboratory, Dayton, Ohio (Received November 13, 1959).

The proton polarization resulting from proton-deuteron elastic scattering has been measured in a double-scattering experiment. The first scattering took place in helium, which served as a polarizer, and the left-right asymmetry observed in a second scattering in deuterium. Spurious asymmetries were checked by substituting xenon and, separately, helium for deuterium as the second scatterer. Measurements taken at a proton-deuteron scattering energy of $E_p = 3.4$ Mev, $\theta = 45°$ and 90°, and $E_p = 3.74$ Mev, $\theta = 45°$ all yielded results consistent with no polarization. From these data it is concluded that the proton polarization in p-d elastic scattering is $< \sim 10\%$ in this energy region.

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NONELASTIC SCATTERING OF FAST NEUTRONS. J. T. Prud'homme, P. L. Okhuysen,* and I. L. Morgan, Texas Nuclear Corporation, Austin, Texas (Received December 2, 1959).

The relative angular distributions of neutrons inelastically scattered from iron, yttrium, zirconium, radiogenic lead (88% Pb^{206}), lead, and bismuth were measured for neutrons in the region from 3.7 to 4.7 Mev. The relative angular distributions of the low-energy (0.5 to 4 Mev) neutrons resulting from nonelastic scattering of 15.2-Mev neutrons were also measured. In each case the distributions were found to be isotropic within experimental error (± 15%), therefore supporting earlier evidence of compound nucleus formation as the predominant interaction mechanism.

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DECAY OF 45-DAY Fe^{59}. R. L. Heath, C. W. Reich, and D. G. Proctor, Atomic Energy Division, Phillips Petroleum Company, Idaho Falls, Idaho (Received December 14, 1959).

The gamma rays following the decay of 45-day Fe^{59} have been studied using the techniques of gamma-ray scintillation spectrometry, including γ-γ coincidence and γ-γ directional correlation measurements. In addition to the previously reported 0.192-, 1.10-, and 1.29-Mev gamma rays, two additional gamma rays having energies of 0.145 and 0.337 Mev were observed. These latter two arise from a state at 1.43 Mev in Co^{59}. All gamma rays were observed to decay with a half-life of 45 ± 5 days. Directional correlation

measurements on the 0.14-1.29 Mev and 0.19-1.10 Mev cascades were performed. The results of these measurements, together with the gamma-ray relative intensities, suggest an assignment of $\frac{1}{2}$- to the 1.43-Mev state in Co^{59}.

SURVEY OF (p,d) REACTIONS AT 22 MEV. C. D. Goodman and J. B. Ball, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received December 11, 1959).

Energy spectra of deuterons from (p,d) reactions on medium and heavy weight elements were surveyed. The experimental method of particle identification is described. The spectra show gross structure indicative of strong selection rules. The gross structure can be correlated with nuclear shell structure, and the levels which are most strongly excited are those which have the same shell configurations as the target with one neutron missing. Angular distributions confirm the shell assignments. This leads to a picture of the reaction mechanism for (p,d) reactions in which the incoming proton interacts principally with a single neutron rather than with the nucleus as a whole.

MOMENTS OF INERTIA OF EVEN-EVEN EARTH NUCLEI. J. J. Griffin and M. Rich, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received November 16, 1959).

Moments of inertia of even-even nuclei are computed using the Nilsson model for deformed nuclei and the moment formula derived from the superconductor theory of nuclei. Values for the energy gap and the deformation are obtained from appropriate experimental data. Good agreement is found between the computed and observed energies of the first excited states of twenty-six rare earth nuclei. This success lends strong support to the superconductor theory of the nucleus.

CHARGE INDEPENDENCE IN THE REACTIONS $p+d\rightarrow\pi^0+He^3$ AND $p+d\rightarrow\pi^+ + H^3$ at 450 Mev. A. V. Crewe,* B. Ledley,† E. Lillethun, S. M. Marcowitz, and C. Rey, The Enrico Fermi Institute for Nuclear Studies, The University of Chicago, Chicago, Illinois (Received December 30, 1959).

An experiment has been performed to measure the branching ratio in the production of He^3 and H^3 in p-d collisions at 450 Mev at 140° in the c.m. system. The result is 2.13 ± 0.15 which is in agreement with the prediction of a ratio 2 on the basis of charge independence alone. The production cross sections were found to be $(d\sigma/d\Omega)_{He^3}=5.41\pm0.29$ μb/sr and $(d\sigma/d\Omega)_{H^3}=11.55\pm0.49$ μb/sr.

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SEARCH FOR AN ELECTRIC DIPOLE MOMENT STRUCTURE OF THE MUON. David Berley and George Gidal, Columbia University, New York, New York (Received December 8, 1959).

A search was made for an electric dipole moment in the muon with a sensitivity of 0.1% of a muon Compton wavelength times the electronic charge. The motivation for this investigation is provided by the interest in finding some property of the muon which would indicate a structure different from that of the electron, even though such a structure would violate both parity conservation and time-reversal invariance. The muons pass through the fringe field of the cyclotron and an additional system of magnets producing an electric field in their rest frame. Any electric dipole moment would precess about this field, producing a vertical plane component of spin transverse to the momentum. The absence of such a component within the stated sensitivity gives an upper limit to the electric dipole moment of the muon as 2×10^{-16} cm $\times$ the charge of the electron.

NULL ELECTROMAGNETIC FIELDS IN GENERAL RELATIVITY THEORY. Asher Peres, Department of Physics, Israel Institute of Technology, Haifa, Israel (Received December 17, 1959).

The existence of null electromagnetic fields, within the frame of the general relativity theory, has been subject to some doubt. We here construct and investigate two solutions of the Maxwell-Einstein equations, representing null electromagnetic fields, and corresponding to unidirectional radiation flow. One of these fields